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*Book **1**.*

015. (2024 AMC 10-A)

A formula used to predict the time required to hike to the top of a mountain trail is given by $T=aL+bG$, where a and b are constants, T represents the time in minutes, L is the trail's length in miles, and G is the elevation gain in feet. According to the model, it takes 69 minutes to hike to the top if the trail is 1.5 miles long with an elevation gain of 800 feet, and also if the trail is 1.2 miles long with an elevation gain of 1100 feet.

How many minutes does the model estimate it will take to hike to the top if the trail is 4.2 miles long and ascends 4000 feet?

STEP 1 ›

A linear model $T=aL+bG$ is used to estimate hiking time, where T is the time in minutes, L is the length of the trail in miles, and G is the altitude gain in feet. Using the model $T=aL+bG$, write a system of two equations:

$$T=aL_1+bG_1 \quad \text{and} \quad T=aL_2+bG_2$$

What are the values of L_1, G_1, L_2, and G_2 that correspond to the two trails described?

A $L_1 = 1.5,\quad G_1 = 800,\quad L_2 = 1.2,\quad G_2 = 1100$

B $L_1 = 1.5,\quad G_1 = 1100,\quad L_2 = 1.2,\quad G_2 = 800$

C $L_1 = 1.5,\quad G_1 = 69,\quad L_2 = 1.2,\quad G_2 = 69$

D $L_1 = 1.2,\quad G_1 = 69,\quad L_2 = 1.2,\quad G_2 = 69$

CONTINUE

STEP 2 ›

Using the result from Step 1, and based on the original model and the data provided, determine the values of the constants a and b.

A $a = 10, \quad b = 0.01$

B $a = 10, \quad b = 0.03$

C $a = 30, \quad b = 0.03$

D $a = 30, \quad b = 0.3$

STEP 3 ›

Using the values of a and b found in Step 2, how many minutes it would take to hike a trail that is 4.2 miles long and ascends 4000 feet.

029. (2023 AMC 10-A)

If a positive integer n can be written as the product of two positive integers a and b, that is, $n=ab$, then a and b are called the complementary divisors of n.
Assume that N is a positive integer which has one pair of complementary divisors whose difference is 20, and another pair of complementary divisors whose difference is 23.

What is the digit sum of N?

STEP 1 >

Two positive integers multiply to form a number N, and their difference is 20. Another pair of positive integers also multiply to give the same number N, and their difference is 23.

Which of the following equations correctly expresses the condition that both pairs of numbers multiply to the same value N?

A $ab + 20 = ab - 23$

B $a(a + 20) = b(b - 23)$

C $a(a + 20) = b(b + 23)$

D $(a + 20)(b - 23) = ab$

E $(a + 23)(b - 20) = ab$

CONTINUE

STEP 2 ›

Using the equation derived in Step 1, what pair of positive integers (a, b) satisfies the equation?

STEP 3 ›

From the result of Step 1 and Step 2, what is the sum of the digits of the common product, N?

A 15

B 16

C 17

D 18

E 19

057. (2024 AMC 10-B)

Balls numbered 1, 2, 3, ... are placed into five bins labeled A, B, C, D, and E according to the following process. Ball 1 goes into bin A, while balls 2 and 3 are placed into bin B. The following 3 balls go into bin C, then the next 4 are placed in bin D, and the process continues in this pattern, returning to bin A after filling bin E. (For example, balls numbered 22, 23, ..., 28 are deposited in bin B at step 7 of this process.)

Which bin contains ball number 2032?

STEP 1 >

Balls are arranged in rows so that the 1st row contains 1 ball, the 2nd row contains 2 balls, the 3rd row contains 3 balls, and so on.
If the number of the last ball in the nth row is expressed in the form

$$\frac{a^x(b^y+c)}{d},$$

what is the value of $a+b+c+d-x-y$?

A $n-2$

B $n+2$

C $2n-1$

D $2n+1$

E $2n+2$

CONTINUE

STEP 2 ›

Based on the formula identified in Step 1 for the last ball in row n, what is the largest value of n such that the last ball in row n is numbered at most 2032?

A 61

B 62

C 63

D 64

E 65

STEP 3 ›

Ball number 2032 is placed following the pattern. In which bin is it deposited?

A bin A

B bin B

C bin C

D bin D

E bin E

058. (2022 AMC 10-B)

Consider a sequence of numbers a_0, a_1, a_2, ... such that each element a_k is either 0 or 1. For each positive integer n, define;

$$S_n = \sum_{k=0}^{n-1} a_k 2^k$$

Given that $7S_n \equiv 1\ (mod\ 2^n)$ for all $n \geq 1$, what is the value of the sum

$$a_{2019} + 2a_{2020} + 4a_{2021} + 8a_{2022}?$$

STEP 1 ›

Let $a_0 = 1$, $a_1 = 0$, $a_2 = 1$, and $a_3 = 1$.
Define

$$S_4 = \sum_{k=0}^{3} a_k \cdot 2^k.$$

What is the value of S_4 ?

A 10

B 11

C 12

D 13

E 14

CONTINUE

STEP 2 >

Suppose that Sn is a positive integer such that

$$7S_n \equiv 1 \pmod{2^n}.$$

Let n=2019.

What is the value of $2^{2019} \pmod{7}$?

A 0

B 1

C 3

D 4

E 6

STEP 3 >

For each positive integer n, define;

$$S_n = \sum_{k=0}^{n-1} a_k\, 2^k$$

Given that $7S_n \equiv 1\ (mod\ 2^n)$ for all $n \geq 1$, what is the value of the sum

$$a_{2019} + 2a_{2020} + 4a_{2021} + 8a_{2022}?$$

079. (2022 AMC 10-B)

Among the first ten terms of the sequence 143, 11433, 1114333, ...,

how many are prime numbers?

STEP 1 >

The first three terms of a sequence are:

$$a_1 = 143, \qquad a_2 = 11433, \qquad a_3 = 1114333.$$

Each term appears to follow a structure.

Which of the following is a correct general expression for the nth term of the sequence?

A $10^n + 3$

B $(10^n + 3)(10^n + 1)$

C $(10^n + 4)(10^n + 3)$

D $10^{n+1} + 4 \cdot 10^n + 3$

E $(10^n + 3)\left(\sum_{k=0}^{n} 10^k\right)$

CONTINUE

STEP 2 ›

Using the result of Step 1,

how many of the first 10 terms of this sequence are prime numbers?

A 0

B 1

C 2

D 3

E 4

087. (2024 AMC 10-A)

A list of 9 real numbers consists of 1, 2.2, 3.2, 5.2, 6.2, and 7, as well as x, y, and z with $x \le y \le z$. The range of the list is 7, and the mean and the median are both positive integers.

How many ordered triples (x, y, z) are possible?

STEP 1 >

In the list of 9 real numbers described above - which includes the six numbers 1, 2.2, 3.2, 5.2, 6.2, and 7, as well as three additional numbers $x \le y \le z$ - if the mean of all 9 numbers must be a positive integer, which of the following could NOT be the value of $x+y+z$?

A 2.2

B 11.2

C 20.2

D 29.2

CONTINUE

STEP 2 ›

Continuing from the previous question:
Assume that the sum of the three unknown numbers $x+y+z$ is one of the values that makes the overall mean a positive integer.

If the list of 9 real numbers must also have a median that is a positive integer, what condition must the middle value y satisfy?

A y must be less than 3.2

B y must be exactly equal to 3.2

C y must be a 5th number in the sorted list

D y must be greater than all six fixed numbers

STEP 3 ›

Given the same list of 9 real numbers as before - including the six fixed values 1, 2.2, 3.2, 5.2, 6.2, and 7, and three additional numbers $x \le y \le z$ - suppose the following two conditions must be satisfied:
the mean and median of the list are both positive integers, and the range of the full list is exactly 7.

How many distinct ordered triples (x, y, z) satisfy all these conditions?

We are **given that $y=\frac{1}{4}$ pound**. Substituting into the equation:

$$x=\frac{36}{5}y \quad \rightarrow \quad \boldsymbol{x}=\frac{36}{5}\left(\frac{1}{4}\right)=\frac{\mathbf{9}}{\mathbf{5}}=\mathbf{1}\frac{\mathbf{4}}{\mathbf{5}}.$$

So,

the weight of **one whole pizza is $1\frac{4}{5}$ pounds**.

Therefore, the correct answer is **A**.

009. *Answer and Explanation*

Step 1.

Let's break it down:
By definition, $\sqrt{(\boldsymbol{x}-\mathbf{1})^{\mathbf{2}}}=|\boldsymbol{x}-\mathbf{1}|$.
This is because the **square root of a square** returns the **absolute value** of the original expression.

Now we need to evaluate $|x-1|$ under the condition $x<0$. **Since $x<0$**, then clearly

$$\boldsymbol{x}-\mathbf{1}<-\mathbf{1}<\mathbf{0} \quad \text{so } \boldsymbol{x}-\mathbf{1} \text{ is negative}.$$

So,

$$|\boldsymbol{x}-\mathbf{1}|=-(\boldsymbol{x}-\mathbf{1})$$

which removes the absolute value by negating the inside (because it was negative).

Therefore, the correct answer is **B**.

Step 2.

From Step 1, we know that:

$$\sqrt{(\boldsymbol{x}-\mathbf{1})^{\mathbf{2}}}=-(\boldsymbol{x}-\mathbf{1})$$

for all $x<0$, because $x-1<0$ in this case.

So substitute into the expression:

$$\left|\boldsymbol{x}-\mathbf{3}-\left(-(\boldsymbol{x}-\mathbf{1})\right)\right|=|x-3+x-1|=|\mathbf{2}\boldsymbol{x}-\mathbf{4}|.$$

Now,
because $\boldsymbol{x}<\mathbf{0}$, then $2x<0$, so $\mathbf{2}\boldsymbol{x}-\mathbf{4}<-\mathbf{4}<\mathbf{0}$, which means the expression inside the absolute value is negative. Hence:

$$|2x-4|=-(\mathbf{2}\boldsymbol{x}-\mathbf{4})=\mathbf{4}-\mathbf{2}\boldsymbol{x}.$$

Therefore, the correct answer is **C**.

Step 3.

Apply the LCM condition:

$$\operatorname{lcm}(n,18)=2^{\max(a,1)}\cdot 3^{\max(b,2)}\cdot 5^{\max(c,0)}=2^2\cdot 3^2\cdot 5.$$

Thus, from the LCM condition: $a=2$, **b=0, 1, or 2**, $c=1$.

Apply the GCD condition:

$$\textbf{gcd}\ (\boldsymbol{n}, \mathbf{45}) = \mathbf{2}^{\mathbf{min}(\boldsymbol{a},\mathbf{0})} \cdot \mathbf{3}^{\mathbf{min}(\boldsymbol{b},\mathbf{2})} \cdot \mathbf{5}^{\mathbf{min}(\boldsymbol{c},\mathbf{1})} = 15 = \mathbf{3^1} \cdot \mathbf{5^1}.$$

Compare exponents:

min (a, 0)=0 → this is valid since $\boldsymbol{a}$**=2**,
min (b, 2)=1 → $\boldsymbol{b}$**=1**,
min (c, 1)=1 → $\boldsymbol{c}$**=1**.

Now we finalize: a=2, b=1, c=1.
So the value of $\boldsymbol{n}$ **is:** $\boldsymbol{n} = \mathbf{2^1} \cdot \mathbf{3^1} \cdot \mathbf{5^1} = \mathbf{60}$.

Therefore, the correct answer is **60**.

010. *Answer and Explanation*

We'll use the first approach and simplify the nested fraction first. Starting from the inside, we have:

$$\frac{1}{3 + \frac{1}{3}} = \frac{1}{\frac{9+1}{3}} = \frac{1}{\frac{10}{3}} = \frac{3}{10}$$

Substituting this into the original expression gives:

$$3 + \frac{1}{3 + \frac{3}{10}} = 3 + \frac{1}{\frac{33}{10}} = 3 + \frac{10}{33} = \frac{99 + 10}{33} = \frac{109}{33}$$

So the value of the expression is $\mathbf{\frac{109}{33}}$.

Therefore, the correct answer is **E**.

011. *Answer and Explanation*

Step 1.

We are given the equation: $\sqrt{\boldsymbol{n}^{\mathbf{2}} - \mathbf{49}} = \boldsymbol{m}$. Our goal is to identify which of the answer choices is equivalent to this equation **after squaring both sides and factoring**.

Eliminate the square root by squaring both sides:

$$\left(\sqrt{n^2 - 49}\right)^2 = m^2 \quad \rightarrow \quad \boldsymbol{n}^{\mathbf{2}} - \mathbf{49} = \boldsymbol{m}^{\mathbf{2}}$$

Rearrange the equation and apply the **difference of squares** identity:

$$n^2 - m^2 = 49 \quad \rightarrow \quad (\boldsymbol{n} + \boldsymbol{m})(\boldsymbol{n} - \boldsymbol{m}) = \mathbf{49}.$$

Therefore, the correct answer is **C**.

Step 2.

We are solving: $\boldsymbol{(n+m)(n-m)=49}$.
Since 49 is a small positive integer, we can list **all integer factor** pairs of 49: **(1,49)**, **(7,7)**, **(-1,-49)**, and **(-7,-7)**.
Let's set: $\boldsymbol{n+m=a}$, $\boldsymbol{n-m=b}$ with $\boldsymbol{ab=49}$.
Then:

$$2n = a + b \quad \rightarrow \quad \boldsymbol{n = \frac{a+b}{2}}$$
$$2m = a - b \quad \rightarrow \quad \boldsymbol{n = \frac{a-b}{2}}$$

Now check that these yield integer values of n and m:

(1, 49): $\boldsymbol{n=25, m=24}$ (7, 7) : $\boldsymbol{n=7, m=0}$
(-1,-49): $\boldsymbol{n=-25, m=-24}$ (-7,-7): $\boldsymbol{n=-7, m=0}$

Thus, there are **4 valid integer pairs** $\boldsymbol{(m,n)}$ that satisfy the original equation.
Therefore, the correct answer is **4** or four.

012. ***Answer and Explanation***

Step 1.

We are given that she has taken $\boldsymbol{n}$ **quizzes** with a **mean score of** $\boldsymbol{m}$. By the definition of mean, this means the total score so far is:

total score=$\boldsymbol{nm}$.

If she **scores 11 on her next quiz**, the new values becomes:

new total = $\boldsymbol{nm+11}$
total number of quizzes = $\boldsymbol{n+1}$

So, the **new mean** is: $\boldsymbol{\frac{nm+11}{n+1}}$.
We are told that this **new mean equals** $\boldsymbol{m+1}$, so we set up the equation:

$$\boldsymbol{\frac{nm+11}{n+1} = m+1.}$$

Therefore, the correct answer is **B**.

Step 2.

We are told that her **total score** for the quizzes she has taken so far is $\boldsymbol{nm}$, and the **number of quizzes is** $\boldsymbol{n}$. She takes **3 more** quizzes, **each scoring 11**, so the new total score becomes:

$nm+11+11+11=\boldsymbol{nm+33}$.

The **new total number** of quizzes is: $\boldsymbol{n+3}$. We are told that this **new mean is** $\boldsymbol{m+2}$. So, we set up the equation using the definition of mean:

$$\boldsymbol{\frac{nm+33}{n+3} = m+2.}$$

Therefore, the correct answer is **A**.

So, the possible **value of k is**;

$$-3, \quad -2, \quad -1, \quad 0, \quad 1, \quad 2.$$

Therefore, the correct answer is **D**.

015. ***Answer and Explanation***

Step 1.

We are given a linear model for estimating hiking time:

$T=aL+bG$ where: $T=$ **time in minutes**
$L=$ **length of the trail in miles**
$G=$ **altitude gain in feet**
$a, b=$ **constants to be determined**.

We are told that both trails take 69 minutes, so we substitute $T=69$ for each case.

① Trail 1: length: $L=1.5$ miles, and elevation gain: $G=800$ feet
Substitute into the model: $\boldsymbol{T=1.5a+800b}$.

② Trail 2: length: $L=1.2$ miles, and elevation gain: $G=1100$ feet
Substitute into the model: $\boldsymbol{T=1.2a+1100b}$.
Therefore, the correct answer is **A**.

Step 2.

From Step 1, we have two equations derived from the model $T=aL+bG$, based on the two trails that **both take 69 minutes** to hike. Let's restate those two equations:

$$\mathbf{69=1.5}\boldsymbol{a}\mathbf{+800}\boldsymbol{b} \qquad (1)$$
$$\mathbf{69=1.2}\boldsymbol{a}\mathbf{+1100}\boldsymbol{b} \qquad (2)$$

We will eliminate a by subtracting the two equations. First, subtract equation (2) from equation (1):

$$(1.5a+800b)-(1.2a+1100b)=69-69$$
$$1.5a-1.2a+800b-1100b=0 \quad \rightarrow \quad \mathbf{0.3}\boldsymbol{a}\mathbf{-300}\boldsymbol{b}\mathbf{=0}$$

Solve for a in terms of b:

$$0.3a-300b=0 \quad \rightarrow \quad \boldsymbol{a}=\frac{300}{0.3}b=\mathbf{1000}\boldsymbol{b}$$

Plug $a=1000b$ into equation (1):

$$\mathbf{69=}1.5(\mathbf{1000}\boldsymbol{b})+800b=1500b+800b\mathbf{=2300}\boldsymbol{b}$$
$$\boldsymbol{b}=\frac{69}{2300}=\frac{3}{100}=\mathbf{0.03}.$$

So,

$$a=1000b \quad \rightarrow \quad \boldsymbol{a}=1000(0.03)=\mathbf{30}.$$

Thus: $\boldsymbol{a}$**=30,** $\boldsymbol{b}$**=0.03**.
Therefore, the correct answer is **C**.

Simplify the numerator: $x+y+3M+6$.
So:

$$\frac{x+y+3M+6}{5}+3=M \quad \rightarrow \quad \frac{x+y+3M+21}{5}=M$$
$$\rightarrow \quad \boldsymbol{x+y+21=2M}$$

Therefore, the correct answer is **D**.

Step 2.

Given:

$$x+y+21=2M \quad \rightarrow \quad \boldsymbol{x+y=2M-21}.$$

Now observe:

$2M$ is always even (since M is an integer),
21 is odd.

Then:

$$\boldsymbol{x+y=\text{even}-\text{odd}=\text{odd}}.$$

So, **$x+y$ must be an odd number**.

Therefore, the correct answer is **A**.

Step 3.

Try the **smallest possible odd value for $x+y$, which is 3**:
Let **x=1, y=2** → **$x+y$=3**.
Then:

$$x+y=2M-21 \quad \rightarrow \quad 3=2M-21 \quad \rightarrow \quad M=12$$
$$\rightarrow \quad \boldsymbol{M+3=15}.$$

So,
the minimum possible value for the unique mode is **15**.

Therefore, the correct answer is **15**.

084. *Answer and Explanation*

Step 1.

Let's assume the mean is 2:

$$\frac{2+2+5+5+6+D}{6}=\frac{\boldsymbol{20+D}}{\boldsymbol{6}}=\boldsymbol{2} \quad \rightarrow \quad 20+D=12$$
$$\rightarrow \quad \boldsymbol{D=-8}$$

But this **contradicts** the condition that **D is a positive integer**. So, even though 2 is in the set twice, it cannot be the mean.

Therefore, the correct answer is **D**.

Step 2.

We are given:

set: $\{2, 2, 5, 5, 6, D\}$ → sum=20+D → **mean** $=\frac{\boldsymbol{20+D}}{\boldsymbol{6}}$.

We are told that this **mean equals one of the elements in the set**. The candidates are: 5, 6, and D.

① mean = 5

$$\frac{20+D}{6}=5 \quad \rightarrow \quad 20+D=30 \quad \rightarrow \quad D=10 \qquad \text{(valid)}.$$

② mean = 6

$$\frac{20+D}{6}=6 \quad \rightarrow \quad 20+D=36 \quad \rightarrow \quad D=16 \qquad \text{(valid)}.$$

③ mean = D

$$\frac{20+D}{6}=D \quad \rightarrow \quad 20+D=6D \quad \rightarrow \quad D=4 \qquad \text{(valid)}.$$

So,
the sum of the valid values of D is: **10+16+4=30**.

Therefore, the correct answer is **C**.

085. *Answer and Explanation*

The sum of the given integers is $2+8+4+4+6+6=30$. Since there are 7 integers in the data set, the average (arithmetic mean) of the data set is $\frac{\mathbf{30+N}}{\mathbf{7}}$ **and the average must be greater than 4**.

There are 3 possible positive integers for N.
1st if the average is 6;

$$\frac{\mathbf{30+N}}{\mathbf{7}}=\mathbf{6} \quad \rightarrow \quad 30+N=42 \quad \rightarrow \quad \boldsymbol{N=12}$$

2nd if the average is 8;

$$\frac{\mathbf{30+N}}{\mathbf{7}}=\mathbf{8} \quad \rightarrow \quad 30+N=56 \quad \rightarrow \quad \boldsymbol{N=26}$$

3rd if the average is N;

$$\frac{\mathbf{30+N}}{\mathbf{7}}=\boldsymbol{N} \quad \rightarrow \quad 30+N=7N \quad \rightarrow \quad \boldsymbol{N=5}$$

So,
the sum of all positive integers is $12+26+5=\mathbf{43}$.

Therefore, the correct answer is **E**.

086. *Answer and Explanation*

The mean of the five numbers is

$$\frac{3+5+7+12+n}{5}=\frac{\mathbf{27+\boldsymbol{n}}}{\mathbf{5}}.$$

There are **three possibilities for the median**: it is either **5**, **7**, or $\boldsymbol{n}$.

Let's consider three cases for the median:
Case 1: **median and mean = 5**.
In this case, $\boldsymbol{n}$ **must be less than 5**, since the median is the middle number in the ordered list. So we have the inequality $n<5$.

Since the mean and the median are equal,

$$\frac{\mathbf{27+n}}{\mathbf{5}}=\mathbf{5} \quad \rightarrow \quad \boldsymbol{n}=\mathbf{-2}$$

The sequence is $\mathbf{-2}$, $\mathbf{3}$, $\mathbf{5}$, $\mathbf{7}$, $\mathbf{12}$, which does have median 5, so this is a **valid solution**.

Case 2: **median and mean =** 7.

In this case, $\boldsymbol{n}$ **must be greater than** $\mathbf{7}$. Since the mean and the median are equal, we have

$$\frac{\mathbf{27+n}}{\mathbf{5}}=\mathbf{7} \quad \rightarrow \quad \boldsymbol{n}=\mathbf{8}.$$

The sequence is $\mathbf{3}$, $\mathbf{5}$, $\mathbf{7}$, $\mathbf{8}$, $\mathbf{12}$, which does have median 7, so this is a **valid solution**.

Case 3: **median and mean =** $\boldsymbol{n}$.

In this case, $\boldsymbol{n}$ **must be greater than 5 and less than 7**. Since the mean and the median are equal, we have

$$\frac{\mathbf{27+n}}{\mathbf{5}}=\boldsymbol{n} \quad \rightarrow \quad \mathbf{4}\boldsymbol{n}=\mathbf{27} \quad \rightarrow \quad \boldsymbol{n}=\mathbf{6.75}$$

However, **this value of** $\boldsymbol{n}$ **does not satisfy the condition** that n is an integer.

So, the **sum of integers** $\boldsymbol{n}$ **is** $\mathbf{-2+8=6}$.

Therefore, the correct answer is **B**.

087. ***Answer and Explanation***

Step 1.

To determine when the **mean of the 9 numbers** is a positive integer, we must first calculate the sum of the 6 given numbers:

1+2.2+3.2+5.2+6.2+7=**24.8**

Let the **sum** of the **three unknown** numbers be $\boldsymbol{S=x+y+z}$. Then the **total sum of all 9** numbers is: **24.8+$\boldsymbol{S}$**

For the **mean** of the 9 numbers to **be an integer**, we require:

$$\frac{24.8+S}{9}=\text{integer} \quad \rightarrow \quad \mathbf{24.8}+\boldsymbol{S}\equiv\mathbf{0}\ (\mathbf{mod\ 9})$$

This means:

$$\mathbf{24.8}+\boldsymbol{S}\equiv\mathbf{0}\ (\mathbf{mod\ 9}) \quad \rightarrow \quad S\equiv 0-24.8\ \ (\text{mod } 9)$$
$$\boldsymbol{S}\equiv\mathbf{-24.8}\ \ (\mathbf{mod\ 9})$$

or equivalently

$$\boldsymbol{S}\equiv(9-6.8)=\mathbf{2.2}\ \ (\mathbf{mod\ 9}).$$

So, S=x+y+z must be of the form: $\boldsymbol{S}$=**2.2**, **11.2**, **20.2**, **29.2**, … .

Let's list possible values of S that make the mean a positive integer and are reasonable for the values of x, y, z:

$$S=2.2\rightarrow \frac{24.8+\mathbf{2.2}}{9}=\mathbf{3}, \qquad S=11.2\rightarrow \frac{24.8+\mathbf{11.2}}{9}=\mathbf{4}$$

$$S=20.2\rightarrow \frac{24.8+\mathbf{20.2}}{9}=\mathbf{5}, \qquad S=29.2\rightarrow \frac{24.8+\mathbf{29.2}}{9}=\mathbf{6}$$