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VOLUME 3

ADVANCED MATH

Build advanced problem-solving skills for high-level SAT Math questions.

- Quadratic Equations
- Quadratic Functions
- Polynomial Functions
- Exponents & Exponential Functions
- Rational Equations & Functions
- Radical Equations & Functions

KEY CONCEPTS

1

Key Concepts 3 **Solving Quadratic Equations**

① **Factoring**:
Rewrite the quadratic equation as a product of two binomials:

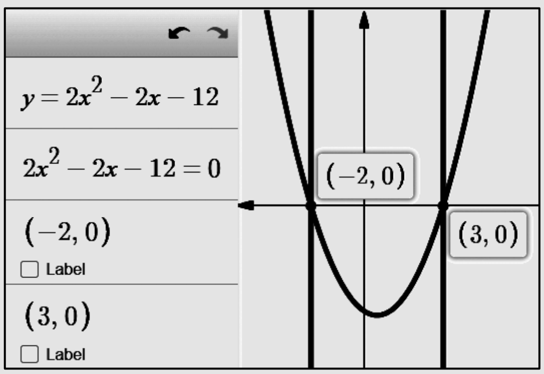
$$ax^2 + bx + c = (\boldsymbol{px + q})(\boldsymbol{rx + s}) = \mathbf{0}$$

Solve for $\boldsymbol{x}$ by setting **each factor** equal to zero:

$$\boldsymbol{px + q = 0} \quad \text{or} \quad \boldsymbol{rx + s = 0}$$

② **Graphical** Method:
The **solutions** to $ax^2 + bx + c = 0$ are the **$\boldsymbol{x}$-intercepts** of the graph of $\boldsymbol{y = ax^2 + bx + c}$.

Use DESMOS:



③ Sum and Product of Roots:
If the **roots** of $ax^2 + bx + c = 0$ are $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$:

sum of roots: $\boldsymbol{\alpha + \beta} = -\dfrac{\mathbf{b}}{\mathbf{a}}$

product of roots: $\boldsymbol{\alpha \cdot \beta} = \dfrac{\mathbf{c}}{\mathbf{a}}$

023.

Given the equation:

$$x^2 - 3x - 18 = 0$$

If b is a solution to the equation above and $b > 0$, what is the value of b?

024.

The function g is defined by $g(x) = x^2 + 9$. For which value of x is $g(x) = 25$?

A) -3

B) 4

C) -5

D) 6

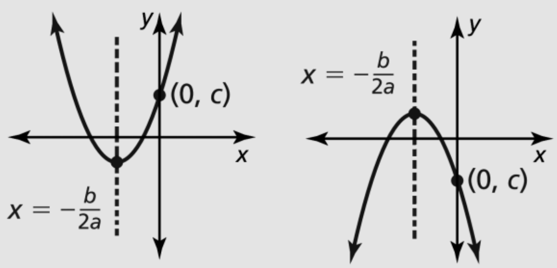
025.

The area of the rectangle is 200 square inches. The length of the rectangle is 5 inches less than 9 times the width of the rectangle. What is the width of the rectangle, in inches?

Key Concepts 2 Standard Form

A quadratic function in **standard form** is written as:

$$f(x) = ax^2 + bx + c$$



① **Vertex**:
The vertex is the **point** (h, k), where:

$$h = -\frac{b}{2a}, \quad k = f(h) = f\left(-\frac{b}{2a}\right)$$

The vertex represents the **maximum** $(a < 0)$ or **minimum** $(a > 0)$ value of the function.

② **Axis of Symmetry**:
The **line** $x = -\frac{b}{2a}$ divides the parabola symmetrically.

③ Intercepts:
y-Intercept:
The **constant term** c is the y-intercept, where $x = 0$.
x-Intercepts:
Solve $ax^2 + bx + c = 0$
to find the x-intercepts (roots).

053.

An ball is kicked from a platform. The equation $h(t) = -4.9t^2 + 11t + 7$ represents this situation, where h is the height of the ball above the ground, in meters, t seconds after it is kicked. What is the height, in meters, from which the ball was kicked?

A) 4

B) 7

C) 13.1

D) 18

054.

x	$f(x)$
-1	9
0	12
2	30

For the quadratic function f, the table shows three values of x and their corresponding values of $f(x)$. Which equation defines f?

A) $f(x) = -x^2 + 4x - 13$

B) $f(x) = 2x^2 + 5x + 12$

C) $f(x) = -3x^2 + 6x - 11$

D) $f(x) = 4x^2 + 7x + 10$

055.

In the xy-plane, the graph of the equation

$$y = -x^2 + 8x - 104$$

intersects the line $y = c$ at exactly one point. What is the value of c?

A) $-\frac{178}{2}$

B) -88

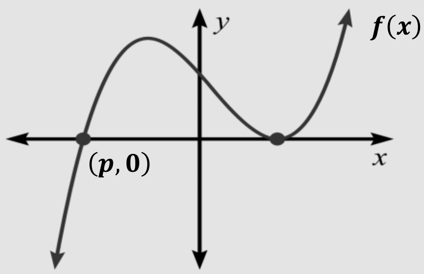
C) -64

D) $-\frac{63}{2}$

03. Polynomial Function

Key Concepts 1 Algebra Principles

① Zeros, Factors, Solutions, and Intercepts:
If $\boldsymbol{f(x)} = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$
$= a(\boldsymbol{x - p})(x-q)(x-r)\ldots.$
then,
A value $\boldsymbol{p}$ **is a zero of** the polynomial $\boldsymbol{f(x)}$.
$\boldsymbol{x - p}$ **is** a **factor of** $\boldsymbol{f(x)}$.
$\boldsymbol{p}$ **is a solution** to the equation $\boldsymbol{f(x) = 0}$.
If $\boldsymbol{p}$ **is a real zero**,
the graph of $f(x)$ **intersects the x-axis at** $(\boldsymbol{p}, \boldsymbol{0})$.



② **Behavior** of the Graph:
The **sign of the leading coefficient** $(\boldsymbol{a_n})$ influences the end behavior of the graph.
The multiplicity of each zero affects how the graph interacts with the x-axis:
Odd multiplicity:
The graph **crosses the x-axis**.
Even multiplicity:
The graph **touches but does not cross** the x-axis.

067.

$$-2(x+3)(x-4)(x-20) = 0$$

Which value is a negative solution to the given equation?

A) -2

B) -3

C) -4

D) -20

068.

The function $g(x) = (x+5)(x-2)(3x+4)$ is defined. Which of the following is NOT an x-intercept of the graph of the function in the xy-plane?

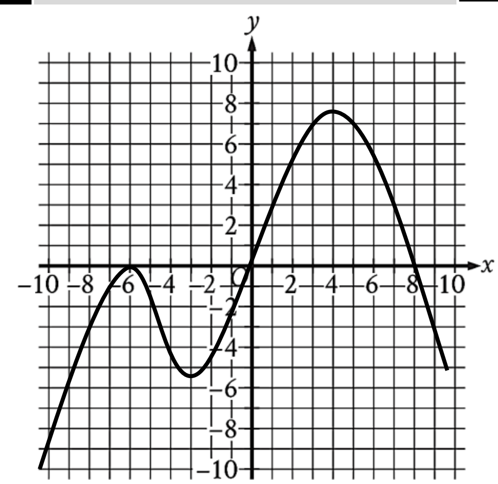
A) $(2, 0)$

B) $(-5, 0)$

C) $\left(\frac{4}{3}, 0\right)$

D) $\left(-\frac{4}{3}, 0\right)$

069.



Which of the following might describe the equation of the shown graph in the xy-plane?

A) $y = \frac{1}{4}x(x-6)(x+8)$

B) $y = -\frac{1}{4}x(x+6)(x-8)$

C) $y = \frac{1}{4}x(x-6)^2(x+8)$

D) $y = -\frac{1}{4}x(x+6)^2(x-8)$

04. Exponents and Exponential Functions

Key Concepts 1 Exponent Rules

① Basic Exponent Rules:
Exponents **follow consistent patterns** that simplify expressions by manipulating powers of the **same base**:

$$a^m \cdot a^n = \boldsymbol{a^{m+n}}$$
$$\frac{a^m}{a^n} = \boldsymbol{a^{m-n}}$$
$$(a^m)^n = \boldsymbol{a^{m\cdot n}}$$
$$(ab)^n = \boldsymbol{a^n b^n}$$
$$\left(\frac{a}{b}\right)^n = \boldsymbol{\frac{a^n}{b^n}}$$

② Additional Rules:
Any nonzero number raised to the power of 0 equals 1:

$$\boldsymbol{a^0 = 1}$$

A **negative exponent** represents the **reciprocal of the base** raised to the positive exponent:

$$\boldsymbol{a^{-n} = \frac{1}{a^n}}$$

③ **Rational Exponents**
Rational exponents are another way to **express roots**:

$$\boldsymbol{a^{\frac{m}{n}} = \sqrt[n]{a^m}} \quad \rightarrow \quad 5^{\frac{2}{3}} = \sqrt[3]{5^2}$$

Combining roots and exponents follows the same rules as integer exponents.

073.

If $\dfrac{\sqrt[3]{x^7}}{\sqrt{x^3}} = x^{\frac{a}{b}}$ for all positive values of x, what is the value of $\dfrac{a}{b}$?

074.

What expression is equal to the $\sqrt[5]{ab^7}$, given that a and b are positive?

A) $a^{-5}b^2$

B) a^5b^7

C) $a^{\frac{1}{5}}b^{\frac{7}{5}}$

D) $a^{\frac{6}{5}}b^{\frac{36}{5}}$

075.

The expression $\dfrac{a^{-3}b^{\frac{1}{3}}}{a^{\frac{1}{2}}b^{-2}}$, where $a > 1$ and $b > 1$, is equivalent to which of the following?

A) $\dfrac{b^3\sqrt{b}}{a^2\sqrt{a}}$

B) $\dfrac{b^2\sqrt{b}}{a^3\sqrt{a}}$

C) $\dfrac{b^3\sqrt[3]{b}}{a^2\sqrt{a}}$

D) $\dfrac{b^2\sqrt[3]{b}}{a^3\sqrt{a}}$

Key Concepts 2 **Radical Equations**

① Definition
Radical equations involve roots, such as square roots ($\sqrt{\ }$) or higher roots.

$$\mathbf{2\sqrt{x+4}-8=0}$$

② Key Steps
Isolate the Radical:
Rearrange the equation so that the **radical is on one side**.

$$2\sqrt{x+4}-8=0$$
$$\rightarrow \quad \mathbf{\sqrt{x+4}=4}$$

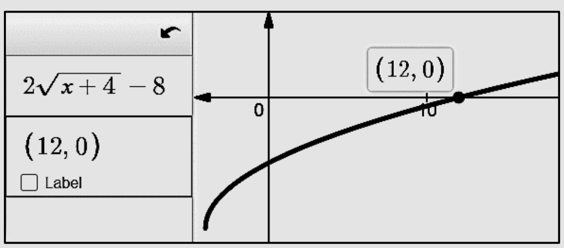
Eliminate the Radical:
Square both sides to remove the square root (or raise to the appropriate power for other roots).

$$\mathbf{\left(\sqrt{x+4}\right)^2=4^2}$$
$$\rightarrow \quad x+4=16$$

Simplify and Solve:
Solve the resulting equation, which may be linear or quadratic.

$$x+4=16$$
$$\rightarrow \quad x=12$$

Or DESMOS:



112.

$$\sqrt{x-5}=12$$

What value of x satisfies the equation above?

113.

$$\sqrt{(x-3)^2}=\sqrt{2x+57}$$

What is the smallest solution to the given equation?

A) 4

B) -4

C) 12

D) -12

114.

$$\frac{x^2}{\sqrt{x^2-a^2}}=\frac{a^2}{\sqrt{x^2-a^2}}+25$$

In the given equation, a is a positive constant. Which of the following is one of the solutions to the equation?

A) $\dfrac{25}{\sqrt{x^2-a^2}}$

B) $\dfrac{a}{\sqrt{x^2+a^2}}$

C) $\sqrt{a^2-25^2}$

D) $-\sqrt{a^2+25^2}$

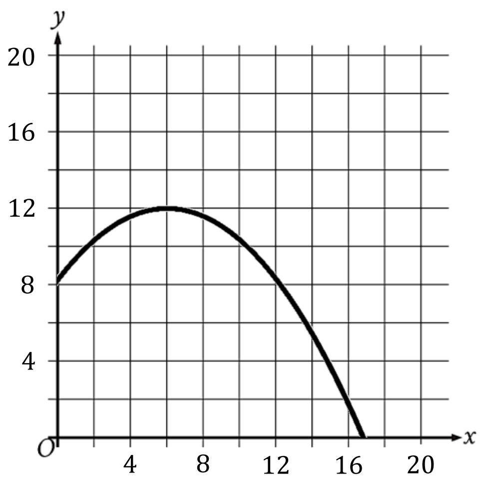
CHALLENGE QUESTIONS

2

155.

In the xy-plane, what is the y-coordinate of the point of intersection of the graphs of $y = (x-2)^2$ and $y = -3x + 6$?

156.



The graph of the function f, defined by

$$f(x) = -\frac{2}{3}(x-6)^2 + 12,$$

is shown in the xy-plane above. If the function g (not shown) is defined by $g(x) = -2x + 12$, what is one possible value of k such that $f(k) = g(k)$?

157.

$$y = x^2 + 3x + 1$$
$$x + y + 2 = 0$$

If $(x_1,\ y_1)$ and $(x_2,\ y_2)$ are the two solutions to the system of equations above, what is the value of $y_1 + y_2$?

A) -2

B) -1

C) 0

D) 1

158.

In a system modeling temperature change, consider the following equations:

$$27x + y = -25$$
$$3x^2 = y + 181$$

The graphs of the equations in the system above intersect at a point (x, y) in the xy-plane. Which of the following is a possible value of x?

A) -12

B) -6

C) 4

D) 13

196.

A stone is thrown vertically from a hill. The quadratic function $h(x) = -16x^2 + 60x + 15$ models the height above the ground h, in meters, of the stone x seconds after it was thrown. If $y = h(x)$ is graphed in the xy-plane, which of the following represents the real-life meaning of the positive x-intercept of the graph?

A) The initial height of the stone.

B) The maximum height of the stone.

C) The time at which the stone hits the ground.

D) The time at which the stone reaches its maximum height.

197.

The kinetic energy (measured in joules) of an object with a mass of 15 kilograms moving at a velocity of v meters per second is represented by the function E, where $E(v) = \frac{15}{2}v^2$. Which of the following is the best interpretation of $E(42) = 13{,}230$ in this context?

A) The object traveling at 42^2 meters per second has a kinetic energy of 13,230 joules.

B) The object traveling at 42 meters per second has a kinetic energy of 13,230 joules.

C) The object traveling at 13,230 meters per second has a kinetic energy of 42 joules.

D) The object traveling at $13{,}230^2$ meters per second has a kinetic energy of 42 joules.

198.

The function $A(w) = 4w^2$ gives the area of a rectangle in square feet (ft^2), where the width is w feet and the length is 4 times the width. Which of the following is the best interpretation of $A(17) = 1{,}156$?

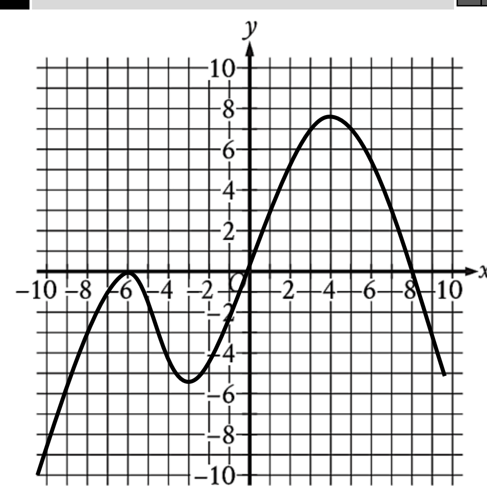
A) If the width of the rectangle is 1,156 ft, then the area of the rectangle is 17 ft^2.

B) If the width of the rectangle is 17 ft, then the area of the rectangle is 1,156 ft^2.

C) If the width of the rectangle is 1,156 ft, then the length of the rectangle is 17 ft.

D) If the width of the rectangle is 17 ft, then the length of the rectangle is 1,156 ft.

199.



Which of the following might describe the equation of the shown graph in the xy-plane?

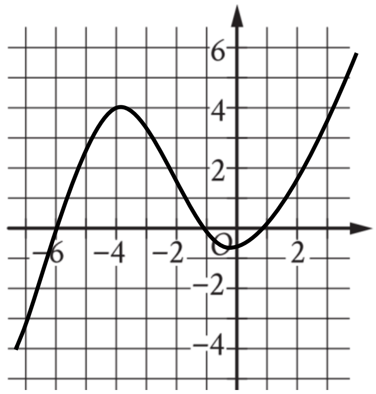
A) $y = \frac{1}{4}x(x-6)(x+8)$

B) $y = -\frac{1}{4}x(x+6)(x-8)$

C) $y = \frac{1}{4}x(x-6)^2(x+8)$

D) $y = -\frac{1}{4}x(x+6)^2(x-8)$

200.



The graph of $y = f(x)$ is shown, where the function f is defined by $f(x) = ax^3 + bx^2 + cx + d$ and a, b, c, and d are constants. For how many values of x does $f(x) = 0$?

A) zero

B) one

C) two

D) three

201.

x	-3	-1	0	1	3
$f(x)$	4	0	-5	-2	0

The table shown provides specific values for the polynomial function f. Given the values in the table, which of the following must be a factor of f?

A) $x(x-1)$

B) $x(x+3)$

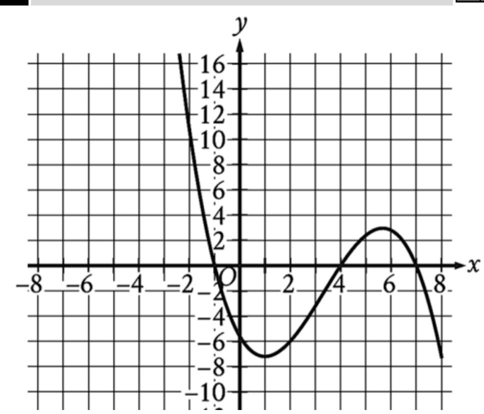
C) $(x-1)(x+3)$

D) $(x+1)(x-3)$

202.

Given the function $f(x) = x^3 + ax^2 + bx + c$, where a, b, and c are integer constants, and if the zeros of the function are -3, 5, and 8, what is the value of c?

203.



The graph of $y = g(x)$ is shown, where the function g is defined by $g(x) = ax^3 + bx^2 + cx + d$ and a, b, c, and d are constants. For how many values of x does $g(x) = 0$?

A) 0

B) 1

C) 2

D) 3

209.

The expression $\dfrac{a^{-3}b^{\frac{1}{3}}}{a^{\frac{1}{2}}b^{-2}}$, where $a > 1$ and $b > 1$, is equivalent to which of the following?

A) $\dfrac{b^3\sqrt{b}}{a^2\sqrt{a}}$

B) $\dfrac{b^2\sqrt{b}}{a^3\sqrt{a}}$

C) $\dfrac{b^3\sqrt[3]{b}}{a^2\sqrt{a}}$

D) $\dfrac{b^2\sqrt[3]{b}}{a^3\sqrt{a}}$

210.

The expression $7\sqrt[6]{5^6x^{48}} \cdot \sqrt[9]{2^9x^3}$ is equivalent to px^q, where p and q are positive constants and $x > 1$. What is the value of $p + q$?

211.

Which of the following is equivalent to $\sqrt[4]{x^2 + 14x + 49}$, where $x > 0$?

A) $(x + 7)^2$

B) $(x + 7)^4$

C) $(x + 7)^{\frac{1}{2}}$

D) $(x + 7)^{\frac{1}{4}}$

212.

If $5^{9k} = \sqrt[4]{5^{18}}$, what is the value of k ?

213.

Which expression is equivalent to

$$x^{\frac{11}{12}},$$

where $x > 0$?

A) $\sqrt[11]{x^{12}}$

B) $\sqrt[12]{x^{132}}$

C) $\sqrt[132]{x^{144}}$

D) $\sqrt[144]{x^{132}}$

218.

$$P = 2{,}300 \cdot (1.04)^t$$

The equation above models the population, P, of a town t years after a conservation program began. Which of the following equations models the population of the town m months after the program began?

A) $P = 2300 \cdot (1.04)^{12m}$

B) $P = 2300 \cdot (1.04)^{\frac{m}{12}}$

C) $P = 2300 \cdot (1.003)^{12m}$

D) $P = 2300 \cdot (1.003)^{\frac{m}{12}}$

219.

A sports center had 4,500 members in 2025. The center predicts that after 2025, the number of members each year will decrease by 23% from the previous year. Which of the following functions represents the relationship between the number of members, $g(x)$, and the number of years after 2025, x?

A) $g(x) = 4{,}500(0.23)^x$

B) $g(x) = 4{,}500(0.77)^x$

C) $g(x) = 4{,}500(1.23)^x$

D) $g(x) = 4{,}500(1.77)^x$

220.

A charity organization initiates a fund with a starting balance of \$25,400.00 . The fund accumulates interest without additional deposits or withdrawals. The total amount in the fund is described by an exponential function A, where $A(t)$ represents the balance in dollars after t years. The fund grows to a balance of \$72,850.46 after 15 years. Which equation could define $A(t)$?

A) $A(t) = 25{,}400(0.073)^t$

B) $A(t) = 72{,}850.46(0.073)^t$

C) $A(t) = 25{,}400(1.073)^t$

D) $A(t) = 72{,}850.46(1.073)^t$

221.

A scientist recorded the temperature of a heated metal rod placed in a lab environment with a stable temperature of 20 degrees Celsius (℃). The rod's temperature was initially 100℃ at 3:00 p.m. when it began to cool. Five minutes later, at 3:05 p.m., the temperature had dropped to 77.6℃, and by 3:10 p.m., it was 61.5℃. The temperature of the rod continued to decline gradually over time. Of the following functions, which best models the temperature $C(t)$, in degrees Celsius, of the metal rod t minutes after cooling started?

A) $C(t) = 20(0.72)^{\frac{t}{5}}$

B) $C(t) = 100(0.72)^{\frac{t}{5}}$

C) $C(t) = 20 + 80(0.72)^{\frac{t}{5}}$

D) $C(t) = 20 + 100(0.72)^{\frac{t}{5}}$

264.

The volume of gas V_G inside a container can be found using the formula

$$V_G = \frac{V_L}{1 - C},$$

where V_L is the liquid volume, and C represents the concentration fraction. Which of the following correctly expresses C in terms of V_G and V_L ?

A) $C = \frac{1 - V_L}{V_G}$

B) $C = \frac{1 + V_L}{V_G}$

C) $C = 1 - \frac{V_L}{V_G}$

D) $C = 1 + \frac{V_L}{V_G}$

265.

$$\frac{4x^2}{x^2 - 9} - \frac{2x}{x + 3} = \frac{1}{x - 3}$$

What value of x satisfies the above equation?

A) -3

B) $\frac{1}{2}$

C) $\frac{3}{2}$

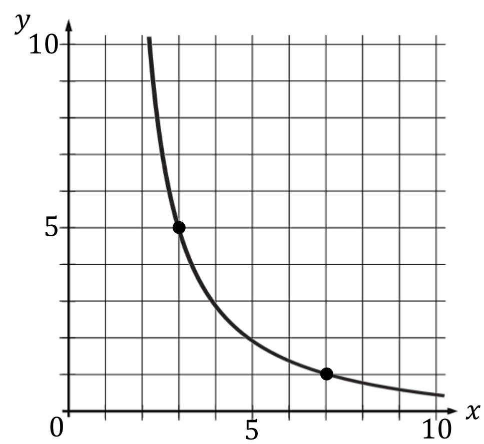
D) $-\frac{5}{4}$

266.

$$\frac{77}{x + 4} = x$$

What is the positive solution to the given equation?

267.



The rational function r is defined by an equation in the form

$$r(x) = \frac{a}{(x + b)},$$

where a and b are constants. A part of the graph of $y = r(x)$ is shown. If $h(x) = r(x + 3)$, which equation could define function h?

A) $h(x) = \frac{5x}{x + 1}$

B) $h(x) = \frac{5}{x + 3}$

C) $h(x) = \frac{5}{x + 1}$

D) $h(x) = \frac{5x}{x + 3}$

268.

The function $f(x)=\frac{p+12}{x-2}+3$ is defined, where p is a constant. The graph of f, in the xy-plane where $y=f(x)$, is shifted 2 units up and 5 units to the left, resulting in the graph of $y=r(x)$. Which equation defines r?

A) $r(x)=\dfrac{p+14}{x+3}+3$

B) $r(x)=\dfrac{p+14}{x-7}+3$

C) $r(x)=\dfrac{p+12}{x+3}+5$

D) $r(x)=\dfrac{p+12}{x-7}+5$

269.

Let the function g be defined as

$$g(x)=\frac{(x-d)^2+150}{3d},$$

where d is a constant. If $g(d)=25$, what is the value of $g(26)$?

A) 96

B) 108

C) 116

D) 121

270.

Given that a and b are positive numbers, which of the following expressions is equal to $\sqrt{(a-b)^3}\cdot\sqrt{a-b}$?

A) $a-b$

B) a^2-b^2

C) $a^2+2ab+b^2$

D) $a^2-2ab+b^2$

271.

$$\sqrt{(x-4)^2}=\sqrt{7x+50}$$

What is the smallest solution to the given equation?

272.

$$\frac{28x}{3y}=4\sqrt{z-25}$$

The given equation relates the distinct positive real numbers x, y, and z. Which equation correctly expresses z in terms of x and y?

A) $z=\dfrac{1}{4}\sqrt{\dfrac{28x}{3y}}+25$

B) $z=\sqrt{\dfrac{7x}{3y}}+25$

C) $z=\dfrac{49x}{9y}+25$

D) $z=\dfrac{49x^2}{9y^2}+25$

273.

$$\frac{x^2}{\sqrt{x^2 - a^2}} = \frac{a^2}{\sqrt{x^2 - a^2}} + 25$$

In the given equation, a is a positive constant. Which of the following is one of the solutions to the equation?

A) $\dfrac{25}{\sqrt{x^2 - a^2}}$

B) $\dfrac{a}{\sqrt{x^2 + a^2}}$

C) $\sqrt{a^2 - 25^2}$

D) $-\sqrt{a^2 + 25^2}$

274.

The function g is defined by $g(x) = p\sqrt{x - q}$, where p and q are constants. In the xy-plane, the graph of $y = g(x)$ passes through the point $(27, 0)$, and $g(31) < 0$. What must be true among the following statements?

A) $p > q$

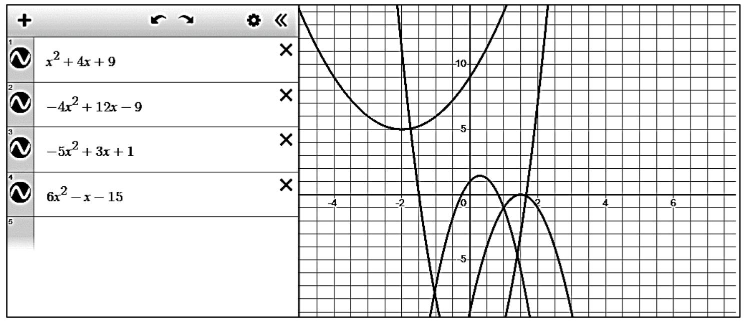
B) $p < q$

C) $g(0) = 27$

D) $g(0) = 31$

Or use DESMOS

$x^2+4x+9=0$	$\Delta = 4^2-4(1)(9) = \mathbf{-20<0}$
$-4x^2+12x-9=0$	$\Delta = 12^2-4(-4)(-9) = \mathbf{0=0}$
$-5x^2+3x+1=0$	$\Delta = (3)^2-4(-5)(1) = \mathbf{29>0}$
$6x^2-x-15=0$	$\Delta = (-1)^2-4(6)(-15) = \mathbf{361>0}$



Therefore, the correct answer is **A**.

040. ***Explanation and Answer***

For a point (x, y) to satisfy a system of equations, both values of x and y must satisfy each equation simultaneously. We can solve this system by **setting the right-hand sides of the equations equal to each othe**r and then solving for x using factoring or graph.

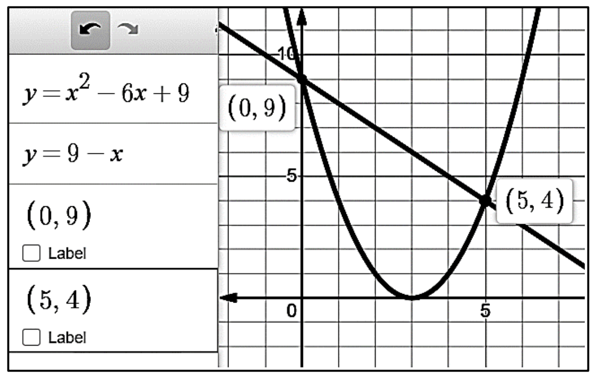
Set the equations equal to each other:
Since both expressions are equal to y,
we can equate them and solve for x:

$$\boldsymbol{x^2-6x+9=9-x} \quad \rightarrow \quad x^2-5x=0$$
$$\rightarrow \quad \boldsymbol{x(x-5)=0}$$

Set each factor equal to zero:

$$x(x-5)=0 \quad \rightarrow \quad \boldsymbol{x=0 \ \ or \ \ x=5}$$

Or use DESMOS:



Therefore, the **one possible value of x is 0 or 5** and the correct answer is **0** or **5**.

041. ***Explanation and Answer***

To find the **solution to the system of equations**, we need to determine where the graphs of the linear equation and the nonlinear equation **intersect**. The **intersection points** of the graphs represent the **solutions to the system**.

The graphs of the linear and nonlinear equations **intersect at the point (3, 4)**. This means that both the linear and nonlinear equations are satisfied **when $x=3$ and $y=4$**.

Therefore, the correct answer is **D**.

042. ***Explanation and Answer***

To solve for x in this system of equations, we start by analyzing the given equations and expressing y in terms of x **using substitution**.

From the equation $x+y=20$, we can express y as:

$$x+y=20 \quad \rightarrow \quad \boldsymbol{y=20-x}$$

Substitute $y=20-x$ into the second equation:

$$xy=91 \quad \rightarrow \quad x(\boldsymbol{20-x})=91 \quad \rightarrow \quad 20x-x^2=91$$

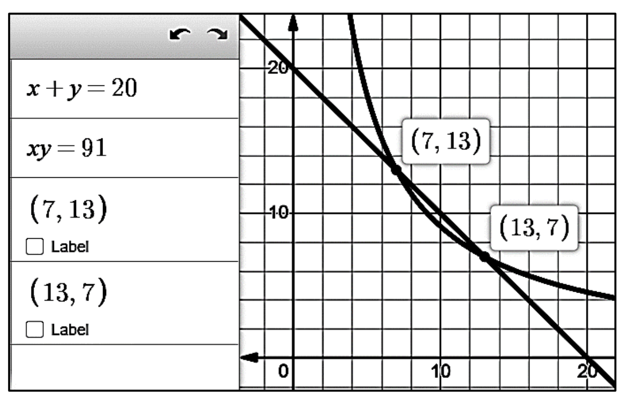
Rearrange it into standard form and factor out:

$$20x-x^2=91 \quad \rightarrow \quad x^2-20x+91=0$$
$$\rightarrow \quad \boldsymbol{(x-13)(x-7)=0}$$

Setting each factor to zero gives:

$$(x-13)(x-7)=0 \quad \rightarrow \quad x-13=0 \quad \rightarrow \quad \boldsymbol{x=13}$$
$$x-7=0 \quad \rightarrow \quad \boldsymbol{x=7}$$

Or use DESMOS



Thus, **one possible value of x is 7**, and the **other possible value is 13**.

Therefore, the correct answer is **7** or **13**.

070. ***Explanation and Answer***

Function **evaluation** involves **substituting the given value** into the function.

We are given $h(x) = x^3 + 17$ and need to find $\boldsymbol{h(3)}$.
Substitute $\boldsymbol{x = 3}$ into the function and evaluate:

$$h(x) = x^3 + 17 \quad \rightarrow \quad h(\boldsymbol{3}) = (\boldsymbol{3})^3 + 17 \quad \rightarrow \quad h(3) = 27 + 17 \quad \rightarrow \quad \boldsymbol{h(3) = 44}$$

Therefore, **the value of** $\boldsymbol{h(3)}$ **is** $\boldsymbol{44}$ and the correct answer is **D**.

071. ***Explanation and Answer***

This problem involves evaluating a polynomial function with specific values for x and understanding transformations by adjusting the function output with a constant.

Given $\boldsymbol{g(x) = (x + 7)(x - 2)(x - 5)}$, we are to find $\boldsymbol{y = g(x) + 4}$ for $x = -7$, $x = 2$, and $x = 5$.

Calculations:

For $\boldsymbol{x = -7}$: $g(x) = (x + 7)(x - 2)(x - 5) \quad \rightarrow \quad g(\boldsymbol{-7}) = (\boldsymbol{-7} + 7)(-7 - 2)(-7 - 5) = \boldsymbol{0}$
so $\boldsymbol{y = g(-7)} + 4 = \boldsymbol{0} + 4 = \boldsymbol{4}$.

For $\boldsymbol{x = 2}$: $g(x) = (x + 7)(x - 2)(x - 5) \quad \rightarrow \quad g(\boldsymbol{2}) = (2 + 7)(\boldsymbol{2} - 2)(-7 - 5) = \boldsymbol{0}$
so $\boldsymbol{y = g(2)} + 4 = \boldsymbol{0} + 4 = \boldsymbol{4}$.

For $\boldsymbol{x = 5}$: $g(x) = (x + 7)(x - 2)(x - 5) \quad \rightarrow \quad g(\boldsymbol{5}) = (5 + 7)(5 - 2)(\boldsymbol{5} - 5) = \boldsymbol{0}$
so $\boldsymbol{y = g(5)} + 4 = \boldsymbol{0} + 4 = \boldsymbol{4}$.

Therefore, for **each value** of $x = -7$, $x = 2$, and $x = 5$, $\boldsymbol{y = 4}$ and the correct answer is **D**.

072. ***Explanation and Answer***

Given information:

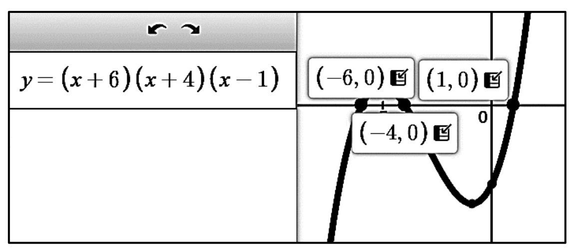
$\boldsymbol{f(x) = (x + 4)(x + 2)(x - 3)}$
$\boldsymbol{r(x) = f(x + 2)}$, which shifts the function horizontally

To find $\boldsymbol{r(x)}$, **replace** $\boldsymbol{x}$ **with** $\boldsymbol{x + 2}$ in each term of $f(x)$:

$$r(x) = f(x + 2) \quad \rightarrow \quad \boldsymbol{r(x)} = (\boldsymbol{x + 2} + 4)(\boldsymbol{x + 2} + 2)(\boldsymbol{x + 2} - 3)$$
$$\rightarrow \quad \boldsymbol{r(x) = (x + 6)(x + 4)(x - 1)}$$

The $\boldsymbol{x}$**-intercepts occur when** $\boldsymbol{r(x) = 0}$. Set each **factor equal to zero**:

$$r(x) = 0 \quad \rightarrow \quad \boldsymbol{(x + 6)(x + 4)(x - 1) = 0}$$
$$\rightarrow \quad x + 6 = 0 \quad \rightarrow \quad \boldsymbol{x = -6}$$
$$x + 4 = 0 \quad \rightarrow \quad \boldsymbol{x = -4}$$
$$x - 1 = 0 \quad \rightarrow \quad \boldsymbol{x = 1}$$



Thus, **the** $\boldsymbol{x}$**-intercepts are** $\boldsymbol{(-6, 0)}$**,** $\boldsymbol{(-4, 0)}$**, and** $\boldsymbol{(1, 0)}$,
so $\boldsymbol{p = -6}$**,** $\boldsymbol{q = -4}$**, and** $\boldsymbol{r = 1}$.
Or use DESMOS:
So, $\boldsymbol{p + q + r = (-6) + (-4) + 1 = -9}$

Therefore, **the sum of** $\boldsymbol{p}$**,** $\boldsymbol{q}$**, and** $\boldsymbol{r}$ **is** $\boldsymbol{-9}$ and the correct answer is **D**.

073. ***Explanation and Answer***

We will begin by rewriting the square root and cube root expressions as rational (fractional) exponents:

$$\boldsymbol{\sqrt[n]{x^m} = x^{\frac{m}{n}}} \quad \rightarrow \quad \boldsymbol{\sqrt[3]{x^7} = x^{\frac{7}{3}}} \qquad \textit{and} \qquad \boldsymbol{\sqrt{x^3} = x^{\frac{3}{2}}}$$

Next, apply the property of exponents that states;

$$x^a \div x^b = \boldsymbol{\frac{x^a}{x^b} = x^{a-b}} \qquad \rightarrow \qquad \frac{x^{\frac{7}{3}}}{x^{\frac{3}{2}}} = x^{\frac{7}{3}-\frac{3}{2}} = \boldsymbol{x^{\frac{5}{6}}}$$

Therefore, $\boldsymbol{\frac{\sqrt[3]{x^7}}{\sqrt{x^3}}}$ **is equivalent to** $\boldsymbol{x^{\frac{5}{6}}}$, which means that $\boldsymbol{\frac{a}{b} = \frac{5}{6}}$ the correct value of is $\boldsymbol{\frac{5}{6}}$.

086. ***Explanation and Answer***

To find the y-intercept of a graph, you need to identify the point where the **graph crosses the y-axis**. This occurs when the x-coordinate is zero. So, the y-intercept is the value of y when x **is 0**.

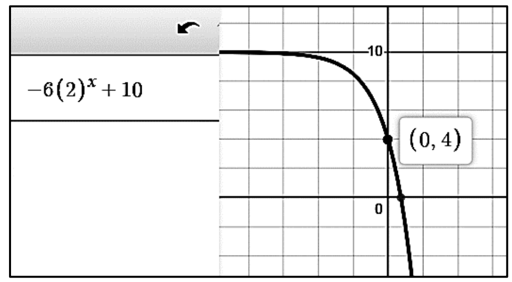
The y-intercept is the point where the graph crosses the y-axis, which **corresponds to the coordinate** $(\mathbf{0}, \boldsymbol{y})$. By observing the graph (as described), the graph crosses the y-axis at the point $(\mathbf{0}, \mathbf{-2})$.
This means that the y-intercept is -2.
Therefore, the correct answer is **B**.

087. ***Explanation and Answer***

The $\boldsymbol{y}$**-intercept** of a graph is the point where the **graph crosses the y-axis**.
This occurs **when $x = 0$**.
Substitute $x = 0$ into the equation:

$$\boldsymbol{g(x) = (-6)(2)^x + 10} \quad \rightarrow \quad g(\mathbf{0}) = (-6)(2)^{\mathbf{0}} + 10$$
$$\rightarrow \quad g(0) = (-6)(\mathbf{1}) + 10$$
$$\rightarrow \quad \boldsymbol{g(0) = 4}$$

So, the y-intercept of the graph of $y = g(x)$ is the point $(\mathbf{0}, \mathbf{4})$.
Or use DESMOS:



Therefore, the correct answer is **A**.

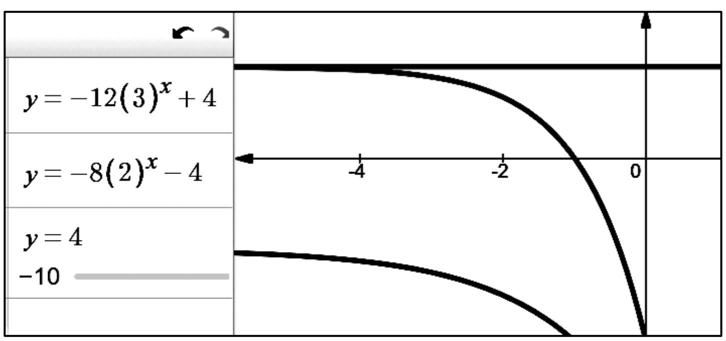
088. ***Explanation and Answer***

Exponential functions of the form $\boldsymbol{a(b)^{x-h} + k}$ have a **horizontal asymptote, $y = k$**, which defines the limit behavior of the function as x approaches positive or negative infinity. A **horizontal asymptote** is a line that the graph of the function **approaches but never touches or crosses**.

The function is $\boldsymbol{f(x) = -12(3)^x + 4}$. This function has a horizontal **asymptote at $y = 4$** because, as $\boldsymbol{x} \rightarrow -\infty$, the exponential term $\mathbf{(3)}^{\boldsymbol{x}}$ **approaches 0**. So, $\boldsymbol{f(x)}$ **approaches 4**. Since the function continues to decrease without bound as x increases (because of the negative coefficient -12), $\boldsymbol{f(x)}$ **does not have a minimum value** but rather approaches the horizontal asymptote without ever reaching it.

The function is $\boldsymbol{g(x) = -8(2)^x - 4}$. For this function, there is a horizontal asymptote at $\boldsymbol{y = -4}$ because, as $x \rightarrow -\infty$, the exponential term $(2)^x$ approaches 0. So, $\boldsymbol{g(x)}$ **approaches -4**. Since the function continues to decrease without bound as x increases (because of the negative coefficient -8), $\boldsymbol{g(x)}$ **does not have a minimum value** but rather approaches the horizontal asymptote without ever reaching it.

Or use DESMOS



Therefore, the correct answer is **D**.

089. ***Explanation and Answer***

The y-intercept of a function graphed as $y = f(t)$ in the ty-plane represents the output value y when $t = 0$.

Calculate $f(0)$: $\boldsymbol{f(0) = 6000 \times (0.75)^0 = 6000 \times 1 = 6000}$
Interpret the y-intercept in context:
The function $f(t) = 6000(0.75)^t$ represents the **number of units distributed by the company** at the end of each year. With $\boldsymbol{t = 0}$ **corresponding to the end of 2025**, $f(0) = 6{,}000$ implies that the company distributed an estimated **6,000 units at the end of 2025**.
Therefore, the correct answer is **A**.

109. ***Explanation and Answer***

To simplify this, recall the property of radicals that states:

$$\sqrt[n]{\boldsymbol{a^n}} = \boldsymbol{a},\ (\boldsymbol{a^n})^{\boldsymbol{m}} = \boldsymbol{a^{n\cdot m}},\ \text{and}\ \sqrt[3]{\boldsymbol{a\cdot b}} = \sqrt[3]{\boldsymbol{a}}\cdot\sqrt[3]{\boldsymbol{b}}$$

So, we can rewrite the expression and simplify each term:

$$\sqrt[3]{\boldsymbol{x^9}} = \sqrt[3]{(\boldsymbol{x^3})^3} = \boldsymbol{x^3}\ , \qquad \sqrt[3]{\boldsymbol{y^6}} = \sqrt[3]{(\boldsymbol{y^2})^3} = \boldsymbol{y^2}$$

Now combine the simplified terms:

$$\sqrt[3]{\boldsymbol{x^9y^6}} = \sqrt[3]{(x^3)^3(y^2)^3} = \boldsymbol{x^3y^2}$$

Therefore, the **simplified expression is x^3y^2** and the correct answer is **C**.

110. ***Explanation and Answer***

First, simplify the square root using $\sqrt{\boldsymbol{a^2}} = \boldsymbol{a}$ and exponent properties $(a^n)^m = a^{n\times m}$:

$$\sqrt{\boldsymbol{36x^8y^6}} = \sqrt{\boldsymbol{6^2(x^4)^2(y^3)^2}} \quad\rightarrow\quad \sqrt{6^2(x^4)^2(y^3)^2} = \sqrt{\boldsymbol{6^2}}\,\sqrt{(\boldsymbol{x^4})^2}\,\sqrt{(\boldsymbol{y^3})^2} = \boldsymbol{6x^4y^3}$$

Now, divide the simplified result by y^4 by using exponent properties $\frac{a^n}{a^m} = a^{n-m}$:

$$\frac{\sqrt{36x^8y^6}}{y^4} = \frac{\boldsymbol{6x^4y^3}}{\boldsymbol{y^4}} = \boldsymbol{6x^4y^{-1}}$$

Therefore, the **expression simplifies to $6x^4y^{-1}$** and the correct answer is **A**.

111. ***Explanation and Answer***

We can use the property of radicals: $\sqrt{\boldsymbol{x}}\cdot\sqrt{\boldsymbol{y}} = \sqrt{\boldsymbol{xy}}$

$$\sqrt{(\boldsymbol{a-b})^3}\cdot\sqrt{(\boldsymbol{a-b})} = \sqrt{(\boldsymbol{a-b})^4}$$

The square root of $\sqrt{(a-b)^4}$ can be simplified using the rule $\sqrt[n]{\boldsymbol{a}} = \boldsymbol{a^{\frac{1}{n}}}$:

$$\sqrt{(a-b)^4} = \left((\boldsymbol{a-b})^4\right)^{\frac{1}{2}} = (\boldsymbol{a-b})^2$$

The result **$(a-b)^2$ is a perfect square**. We can now apply the formula for expanding a binomial square:

$$(\boldsymbol{a-b})^2 = \boldsymbol{a^2 - 2ab + b^2}$$

Therefore, the expression **simplifies to $a^2 - 2ab + b^2$** and the correct answer is **D**.

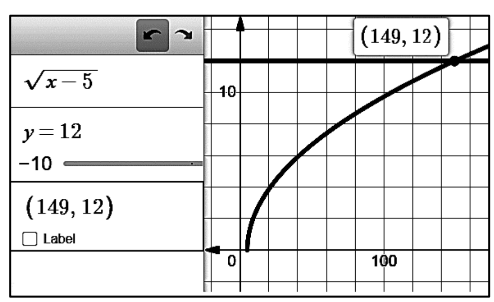
112. ***Explanation and Answer***

We are given: $\sqrt{x-5} = 12$

To **eliminate the square root**, **square both sides** of the equation and solve for x:

$$\sqrt{x-5} = 12 \quad\rightarrow\quad \left(\sqrt{\boldsymbol{x-5}}\right)^2 = \boldsymbol{12^2}$$
$$\rightarrow\quad x - 5 = 144$$
$$\rightarrow\quad \boldsymbol{x = 149}$$

Or use DESMOS



Therefore, **the value of x is 149** and the correct answer is **149**.