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VOLUME 5

GEOMETRY & TRIGONOMETRY

This volume strengthens geometric reasoning and trigonometric problem-solving skills essential for SAT Math.

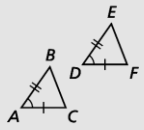
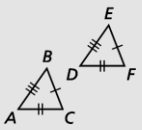
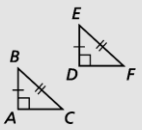
- Geometry
- Circles
- Trigonometry
- Measurement

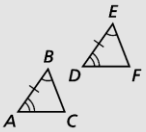
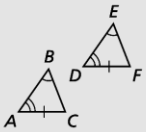
KEY CONCEPTS

1

Key Concepts 3 Triangle Congruency

① Congruency Rules

SAS	SSS	HL (right △ only)
		
Two sides and the included angle are congruent.	All three sides are congruent.	The hypotenuse and one of the legs are congruent.

ASA	AAS
	
Two angles and the included side are congruent.	Two angles and a non-included side are congruent.

② CPCTC:

Corresponding **P**arts of **C**ongruent **T**riangles are **C**ongruent

If **triangles are congruent**, then
all corresponding sides and angles are **equal**.

015.

Triangles △GHI and △JKL are congruent, where G corresponds to J, and angles H and K are right angles. The measure of angle G is 22°. What is the measure of angle L?

A) 22

B) 42

C) 68

D) 158

016.

In triangles ABC and PQR, the measures of angles B and Q are both 36°, and the measures of angles C and R are both 53°. What additional information is sufficient to determine whether triangle ABC is congruent to triangle PQR?

A) No additional information is necessary.

B) the length of side PQ

C) the measure of angle P

D) the lengths of sides BC and QR

017.

In triangles ABC and DEF, angles B and E each measure 32°, and angles C and F each measure 48°. What further information is needed to confirm if triangle ABC is congruent to triangle DEF?

A) The length of side EF.

B) The measure of angle F.

C) The lengths of sides BC and EF.

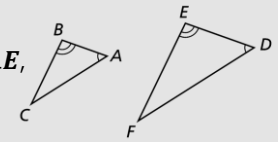
D) No additional information is necessary.

Key Concepts 4 Similarity in Geometry

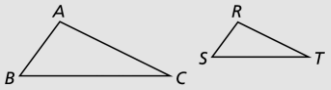
Similarity between two geometric figures implies they have the **same shape but not necessarily the same size**. In similar triangles, the **corresponding angles are equal**, and the **corresponding sides are proportional**.

① Similarity Rules for Triangles

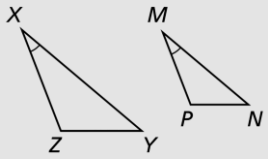
- **AA**
If $\angle \boldsymbol{A} \cong \angle \boldsymbol{D}$ and $\angle \boldsymbol{B} \cong \angle \boldsymbol{E}$, then $\triangle ABC \sim \triangle DEF$.



- **SSS**
If $\frac{\boldsymbol{AB}}{\boldsymbol{RS}} = \frac{\boldsymbol{BC}}{\boldsymbol{ST}} = \frac{\boldsymbol{CA}}{\boldsymbol{TR}}$, then $\triangle ABC \sim \triangle RST$.



- **SAS**
If $\angle \boldsymbol{X} \cong \angle \boldsymbol{M}$ and $\frac{\boldsymbol{ZX}}{\boldsymbol{PM}} = \frac{\boldsymbol{XY}}{\boldsymbol{MN}}$, then $\triangle XYZ \sim \triangle MNP$.



② Key Formulas and Properties

Proportionality of Sides:
In similar triangles, $\triangle ABC \sim \triangle DEF$

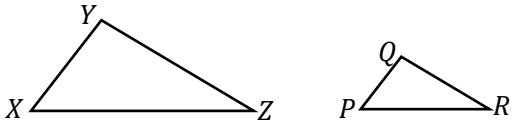
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}.$$

Angles of Similar Triangles:
If $\triangle ABC \sim \triangle DEF$,
$\angle A = \angle D$, $\angle B = \angle E$, and $\angle C = \angle F$.

Scaling Factor:
For **two similar triangles**,
the **ratio of their areas** is the **square** of the ratio of their corresponding sides:

$$\textbf{area ratio} = \left(\frac{\textbf{side}_1}{\textbf{side}_2}\right)^2$$

018.



In triangles $\triangle$ XYZ and $\triangle$ PQR, the sides are proportional such that

$$\frac{XY}{PR} = \frac{YZ}{QP} = \frac{XZ}{QR} = 4.$$

If the measure of angle XYZ is 96°, what is the measure, in degrees, of angle PQR? (Disregard the degree symbol when gridding your answer.)

019.

Triangle ABC is similar to triangle DEF, with vertices A, B, and C corresponding to vertices D, E, and F, respectively. The measure of $\angle C$ is 15°, and $3AB = DE$. What is the measure of $\angle F$?

A) 15°

B) 30°

C) 45°

D) 60°

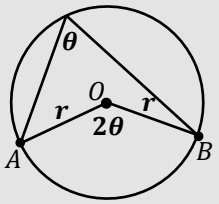
020.

In the right triangle PQR, where angle R is the right angle, the length of QR is 147. Point X on side PQ is joined to point Y on side PR by line segment XY, which is parallel to side QR, with RY being twice the length of PY ($RY = 2PY$). What is the length of line segment XY ?

02. Circle

Key Concepts 1 Arcs and Angles

① **Central Angle** (2θ):
A central angle is **formed by two radii** of a circle, and its measure equals the measure of the intercepted arc.



② **Inscribed Angle** (θ):
An inscribed angle has its **vertex on the circle** and intercepts an arc. Its measure is **half** the measure of the intercepted arc.

③ Arc Length $(\widehat{AB})$:
The length of an arc is calculated using the formula:

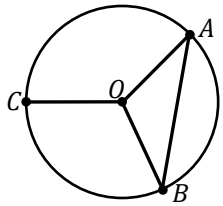
$$\textbf{arc length} = \frac{\textbf{central angle}}{\textbf{360°}} \times \textbf{circumference}$$
$$= \frac{2\theta°}{360°} \times 2\pi r$$

The **circumference of a circle is $2\pi r$**, where r is the radius.

028.

A circle has its center at O, with points A and B positioned on the circle. In triangle OAB, the measure of $\angle AOB$ is 82°. What is the measure of $\angle OAB$, in degrees?

029.



Given that point O is the center of the circle and $\angle OAB$ measures 15°, with the length of $\overline{OC}$ being 24, what is the length of arc $\widehat{AB}$?

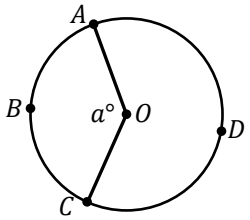
A) 12π

B) 16π

C) 20π

D) 24π

030.



The circle shown has its center at O, with arc ABC measuring 7π, and $a = 140$. What is the length of arc ADC?

A) 11π

B) 14π

C) $\frac{22}{7}\pi$

D) $\frac{36}{7}\pi$

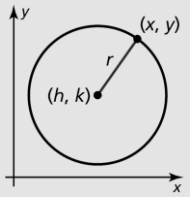
Key Concepts 2 Standard Form of a Circle

① Standard Form Circle Equation:

The **equation of a circle** in the xy-plane is given by:

$$(x-h)^2+(y-k)^2=r^2$$

Where:
$(h,\ k)$ **is the center** and
r **is the radius** of the circle.



If the circle passes through a point $(x_1,\ y_1)$, the radius can be calculated using the distance formula (Pythagorean theorem):

$$r=\sqrt{(x_1-h)^2+(y_1-k)^2}.$$

② **General Form Equation**:
A circle can also be written in general form:

$$ax^2+ay^2+bx+cy+d=0$$

To convert from the standard form by the complete square formula.

$$a^2\pm 2ab+b^2=(a\pm b)^2$$

031.

In the xy-plane, a circle with radius 6 has center $(4,-5)$. What is the equation of the circle from the options below?

A) $(x+4)^2+(y-5)^2=6$

B) $(x+4)^2+(y-5)^2=36$

C) $(x-4)^2+(y+5)^2=6$

D) $(x-4)^2+(y+5)^2=36$

032.

A circle in the xy-plane has a diameter with endpoints $(3,1)$ and $(3,-11)$. An equation of this circle is $(x-3)^2+(y+5)^2=r^2$, where r is a positive constant. What is the value of r?

A) $\sqrt{5}$

B) $\sqrt{11}$

C) 5

D) 6

033.

A circle in the xy-plane has its center at $(5,-4)$ and the point $(9,-9)$ lies on the circle. What equation represents this circle?

A) $(x+5)^2+(x-4)^2=\sqrt{41}$

B) $(x+5)^2+(x-4)^2=41$

C) $(x-5)^2+(x+4)^2=\sqrt{41}$

D) $(x-5)^2+(x+4)^2=41$

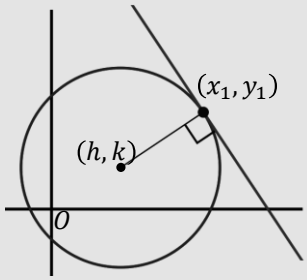
Key Concepts 3 Tangents to a Circle

① Definition:
A tangent is a line that touches a circle at **exactly one point**, called the point of tangency.

② Properties of a Tangent:
The tangent line **is perpendicular to** the radius drawn to the point of tangency.
If the **slope of the radius** is $\boldsymbol{m_r}$, the **slope of the tangent**, $\boldsymbol{m_t}$, is the negative reciprocal:

$$\boldsymbol{m_t = -\frac{1}{m_r}}.$$

③ Formula for Tangent Line



$$\boldsymbol{m_r = \frac{y_1 - k}{x_1 - h}}$$

The **equation of the tangent line** at point (x_1, y_1) can be written as:

$$\boldsymbol{y - y_1 = m_t(x - x_1)},$$

where $m_t = -\frac{1}{m_r}$.

038.

In the xy-plane, there is a circle with its center located at $(2, -1)$. Line l is tangent to this circle at the point $(6, 2)$. Which of the following points also lies on line l ?

A) $(1, 2)$

B) $(-2, 4)$

C) $(3, 6)$

D) $(-4, 8)$

039.

A circle in the xy-plane has its center at $(-7, -5)$. Line t is tangent to this circle at the point $(-9, -9)$. What is the slope of line t ?

A) $\frac{1}{2}$

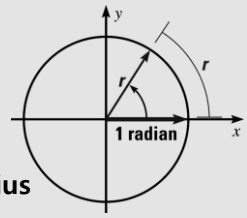
B) $-\frac{1}{2}$

C) 2

D) -2

Key Concepts 4 Degree and Radian Measure

① **Definition** of Radian:
A radian is a unit of angle measure based on the radius of a circle. **One radian** is the angle subtended by an **arc** whose **length equals the radius** of the circle.



② **Conversion** Between Degrees and Radians:

$$\mathbf{180°} = \boldsymbol{\pi} \text{ radians}$$

To convert from degrees to radians:

$$\textbf{radians} = \textbf{degrees} \times \frac{\boldsymbol{\pi}}{\mathbf{180°}}$$

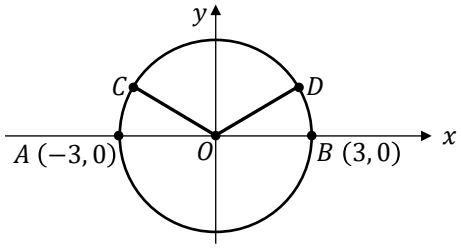
To convert from radians to degrees:

$$\textbf{degrees} = \textbf{radians} \times \frac{\mathbf{180°}}{\boldsymbol{\pi}}$$

040.

An angle has a measure of $\frac{11\pi}{30}$ radians. What is the measure of the angle in <u>degrees</u>?

041.



In the given xy-plane, points A, B, C, and D are located on the circle with its center at O. The angles AOC and BOD both have degree measures of 20°. What is the radian measure of angle COD ?

A) $\frac{3\pi}{7}$

B) $\frac{4\pi}{7}$

C) $\frac{5\pi}{9}$

D) $\frac{7\pi}{9}$

042.

The measure of angle A is $\frac{3\pi}{4}$ radians. The measure of angle B is $\frac{7\pi}{12}$ radians greater than the measure of angle A. What is the measure of angle B, in <u>degrees</u>?

A) 240

B) 285

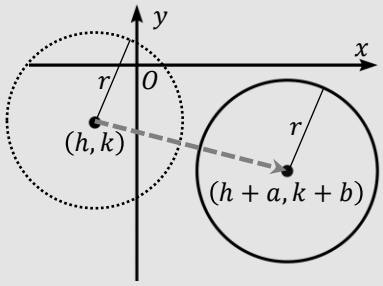
C) 310

D) 330

Key Concepts 5 Translation of a Circle

① Definition:
A translation **shifts** a circle in the xy-plane without altering its size or shape. The **new circle** has the **same radius** but a **different center**.

② Effect on the Circle's Equation:
If the **original circle** is given by
$$(x-h)^2+(y-k)^2=r^2,$$
and the circle is **translated by a units horizontally** and **b units vertically**.



The new center becomes $(h+a, k+b)$. The new equation is:
$$\big(x-(h+a)\big)^2+\big(y-(k+b)\big)^2=r^2.$$

043.

The equation $(x-1)^2+(y+1)^2=64$ represents circle X. Circle Y is obtained by shifting circle A 4 units to the left and 5 units up in the xy-plane. Which of the following equations represents circle Y?

A) $(x+5)^2+(y-4)^2=64$

B) $(x-5)^2+(y+4)^2=64$

C) $(x-3)^2+(y+4)^2=64$

D) $(x+3)^2+(y-4)^2=64$

044.

The equation $(x-2)^2+(y+1)^2=81$ represents circle P. Circle Q is obtained by shifting circle P down 3 units in the xy-plane. Which of the following equations represents circle Q?

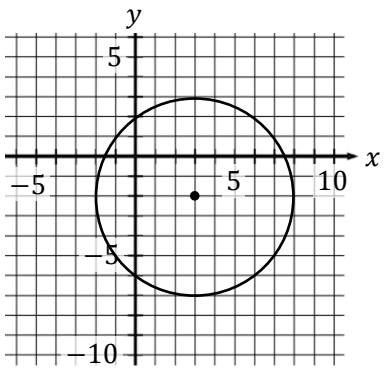
A) $(x-2)^2+(y-1)^2=81$

B) $(x-2)^2+(y+4)^2=81$

C) $(x-4)^2+(y+1)^2=81$

D) $x^2+(y+1)^2=81$

045.



The equation $(x-3)^2+(y+2)^2=100$ represents circle P, which is shown in the diagram. Circle Q (not shown) is the result of shifting circle P up 3 units and decreasing the radius so that the radius of circle Q is half of the radius of circle P. What is the equation that defines circle Q?

A) $(x-3)^2+(y+5)^2=25$

B) $(x-3)^2+(y-1)^2=25$

C) $(x-3)^2+(y-1)^2=50$

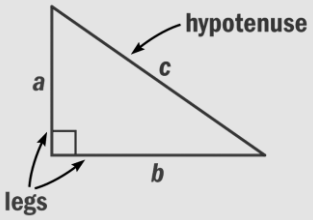
D) $(x-3)^2+(y+5)^2=50$

03. Trigonometry

Key Concepts 1 Right Triangle Property

Right triangles have unique relationships between their angles and side lengths. The Pythagorean Theorem forms the foundation for analyzing these triangles.

① **Pythagorean Theorem**:
For a right triangle with **legs a and b**, and **hypotenuse c**:



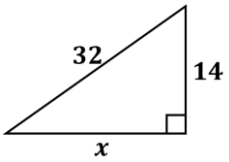
$$c^2 = a^2 + b^2 \quad \rightarrow \quad c = \sqrt{a^2 + b^2}$$
$$a^2 = c^2 - b^2 \quad \rightarrow \quad a = \sqrt{c^2 - b^2}$$

② Applications and **Tips**:
Always verify whether the triangle satisfies the **Pythagorean relationship** when dealing with **right triangles**.

For **inscribed rectangles**, use the **diagonal** relationship to calculate properties of the rectangle and the circle.

Recognize integer triples such as **3, 4, 5** or scaled versions like **6, 8, 10** to quickly solve problems.

046.



Which equation represents the relationship among the side lengths of the given triangle?

A) $14x = 32$

B) $14 + x = 32$

C) $14^2 + x^2 = 32^2$

D) $\sqrt{14 + x} = 32$

047.

Angle B in triangle ABC is a right angle. The side AB measures $20\sqrt{41}$, while side BC measures $21\sqrt{41}$. What is the length of side AC ?

A) 29

B) 31

C) $29\sqrt{41}$

D) $31\sqrt{41}$

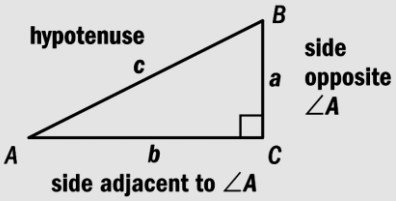
048.

A circle has a rectangle inscribed in it, with each vertex of the rectangle lying on the circle's circumference. The diagonal of the rectangle is twice the length of the shortest side of the rectangle. The area of the rectangle is $1{,}225\sqrt{3}$ square units. What is the length, in units, of the diameter of the circle?

Key Concepts 3 **Trigonometric Ratios**

Trigonometric ratios describe relationships between the angles and side lengths of right triangles.

① **Trigonometric Ratios**



$$\textbf{s}\text{in(A)} = \frac{\textbf{O}\text{pposite side}}{\textbf{H}\text{ypotenuse}} = \frac{\boldsymbol{a}}{\boldsymbol{c}}$$

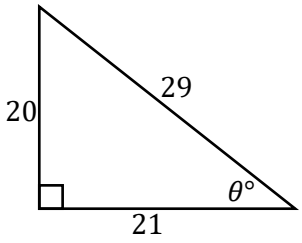
$$\textbf{c}\text{os(A)} = \frac{\textbf{A}\text{djacent side}}{\textbf{H}\text{ypotenuse}} = \frac{\boldsymbol{b}}{\boldsymbol{c}}$$

$$\textbf{t}\text{an(A)} = \frac{\textbf{O}\text{pposite side}}{\textbf{A}\text{djacent side}} = \frac{\boldsymbol{a}}{\boldsymbol{b}}$$

② **Complementary Angle** Relationships
For right triangles, **complementary angles** $(\mathbf{90°} - \boldsymbol{A})$ have special relationships:

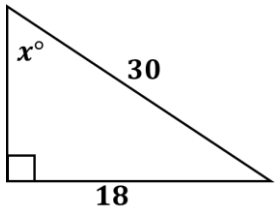
$$\mathbf{sin}(\boldsymbol{A}) = \mathbf{cos}(\mathbf{90°} - \boldsymbol{A}) = \mathbf{cos}(\boldsymbol{B})$$
$$\mathbf{cos}(\boldsymbol{A}) = \mathbf{sin}(\mathbf{90°} - \boldsymbol{A}) = \mathbf{sin}(\boldsymbol{B})$$

052.



In the triangle shown, what is the value of $\cos\theta°$?

053.



In the triangle shown, what is the value of $\sin x°$?

054.

In right triangle ABC, the sum of the measures of angle A and angle B is 90°. The value of $\sin(A)$ is $\frac{\sqrt{11}}{5}$. What is the value of $\cos(B)$?

A) $\frac{\sqrt{6}}{5}$

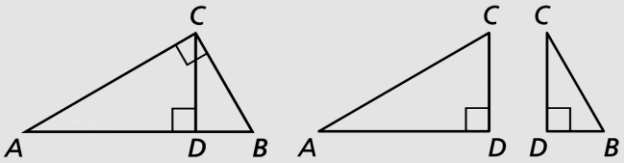
B) $\frac{\sqrt{11}}{5}$

C) $\frac{\sqrt{14}}{5}$

D) $\frac{4}{5}$

Key Concepts 4 **Right Triangle Similarity**

If two triangles are similar, their **corresponding angles are equal**, and their corresponding **sides are proportional**.



① **Two right triangles are similar** if:
One acute angle in one triangle **is equal to** an acute angle in the other triangle.

△CBD ~ △ABC, △ACD ~ △ABC, and △CBD ~ △ACD.

② **Trigonometric Ratio Preservation**:
If two right triangles are similar:
sin, cos, tan values for corresponding acute angles **are the same**.

$$\sin(\theta_1) = \sin(\theta_2)$$
$$\cos(\theta_1) = \cos(\theta_2)$$
$$\tan(\theta_1) = \tan(\theta_2)$$

059.

Triangle XYZ is similar to triangle PQR, where angle X corresponds to angle P, and angles Y and Q are right angles. If $\sin(X) = \frac{209}{241}$, what is the value of $\sin(P)$?

A) $\frac{120}{241}$

B) $\frac{120}{209}$

C) $\frac{209}{241}$

D) $\frac{241}{209}$

060.

$$XY = 15$$
$$YZ = 36$$
$$XZ = 39$$

The side lengths of right triangle XYZ are given. Triangle XYZ is similar to triangle PQR, where Y corresponds to Q and Z corresponds to R. What is the value of $\tan R$?

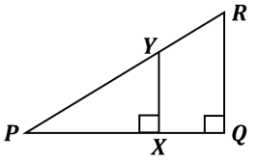
A) $\frac{5}{12}$

B) $\frac{5}{13}$

C) $\frac{7}{12}$

D) $\frac{7}{13}$

061.



In the figure above, $\tan P = \frac{5}{12}$. If $PR = 39$ and $XQ = 12$, what is the length of XY?

CHALLENGE QUESTIONS

2

070.

In triangle DEF, the measure of angle D is 55°. If triangle DEF is isosceles, which of the following is NOT a possible measure of angle E?

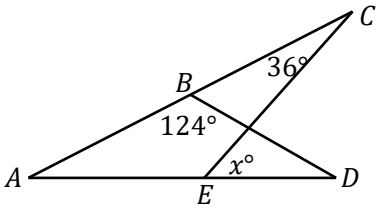
A) 55°

B) 62.5°

C) 70°

D) 110°

071.



In the figure above $AB = BD$. What is the value of x?

A) 28°

B) 48°

C) 64°

D) 86°

072.

In triangles ABC and DEF, angles B and E each measure 32°, and angles C and F each measure 48°. What further information is needed to confirm if triangle ABC is congruent to triangle DEF?

A) The length of side EF.

B) The measure of angle F.

C) The lengths of sides BC and EF.

D) No additional information is necessary.

073.

In triangles XYZ and PQR, angles X and P each have a measure of 45°, $XZ = 12$, and $PQ = 36$. Which additional piece of information is sufficient to prove that triangle XYZ is similar to triangle PQR?

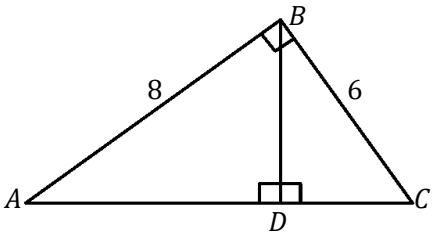
A) $YZ = 8$ and $QR = 8$.

B) $YZ = 8$ and $QR = 24$.

C) The measures of angles Y and Q are 80° and 45°, respectively.

D) The measures of angles Y and Q are 80° and 55°, respectively.

074.



In the figure above, what is the length of $\overline{BD}$?

A) 3.6

B) 4.0

C) 4.4

D) 4.8

075.

At a specific time and location, a tall flagpole casts a shadow that is 240 feet long. At the same moment, a nearby lamp post casts a shadow that is 12 feet long. If the flagpole is approximately 400 feet tall, which of the following is closest to the height, in feet, of the lamp post?

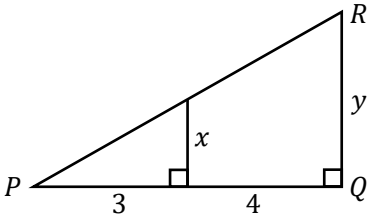
A) 18

B) 20

C) 22

D) 24

076.

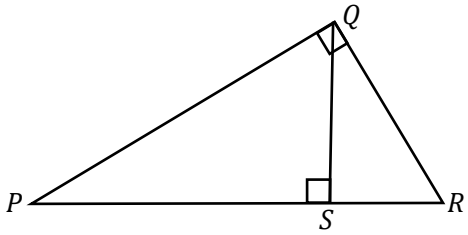


If the area of $\triangle$PQR is between 28 and 35, and y is an integer, find one possible value of x.

077.

In triangle $\triangle$XYZ, angle Z is a right angle. Point M lies on $\overline{XY}$, and point N lies on $\overline{YZ}$ such that $\overline{MN}$ is parallel to $\overline{XZ}$. Given that the length of $\overline{XZ}$ is 78 units, the length of $\overline{MN}$ is 26 units, and the area of triangle $\triangle$XYZ is 936 square units, what is the length of $\overline{YN}$ in units?

078.



In the figure above, right triangle PQR has a right angle at Q, and QS is drawn perpendicular to PR. If $QS = 6$ and $PS = 10$, what is the length of RS?

079.

Pentagons $ABCDE$ and $FGHIJ$ are similar. Each side length of $FGHIJ$ is 7 times that of the corresponding side of $ABCDE$. The pentagon $ABCDE$ has an area of 56 square units. What is the area, in square units, of $FGHIJ$?

A) 392

B) 784

C) 2,744

D) 3,672

085.

A right rectangular prism has a height of 11 inches. The width of the prism's base is x inches, which is 3 inches less than the length of the prism's base. What is the function V that represents the volume of the prism, in cubic inches, based on the width of its base?

A) $V = 11x(x + 3)$

B) $V = 11x(x - 3)$

C) $V = x(x + 11)(x - 3)$

D) $V = x(x - 11)(x + 3)$

086.

A cube has an edge length of 64 inches. Inside the cube is a solid sphere with a radius of 32 inches, positioned so that it touches the center of each face of the cube. What is the volume, to the nearest cubic inch, of the space in the cube that is not occupied by the sphere?

A) 124,886

B) 137,258

C) 146,335

D) 152,913

087.

$$p(x) = 4x + 320$$

The given function p represents the perimeter, in inches (in), of a rectangle with a length of 2 x meters and a fixed width. What is the width, in inches, of the rectangle?

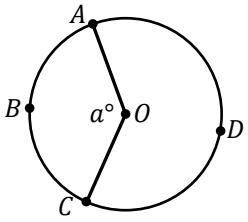
A) 80

B) 120

C) 160

D) 320

088.



The circle shown has its center at O, with arc ABC measuring 7π, and $a = 140$. What is the length of arc ADC?

A) 11π

B) 14π

C) $\frac{22}{7}\pi$

D) $\frac{36}{7}\pi$

ANSWERS & EXPLANATIONS

3

011. ***Explanation and Answer***

In any triangle, the **sum of the measures of the interior angles is 180°**. Since this is a right triangle, one of its angles measures 90°, leaving the remaining two angles to sum to 90° as well.

Since the **two acute angles in a right triangle are complementary** (their measures **add up to 90°**), if one of the acute angles measures 41°, then the other acute angle must also measure:

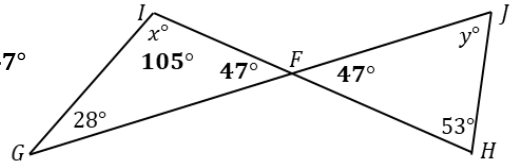
$$\mathbf{90° - 41° = 49°}$$

Therefore, the correct answer is **C**.

012. ***Explanation and Answer***

Use the triangle sum theorem:
Substitute $\boldsymbol{x = 105}$ and solve for x:

$$\mathbf{28° + x° + \angle GFI = 180°} \quad \rightarrow \quad 28° + \mathbf{105°} + \angle GFI = 180°$$
$$\rightarrow \quad 133° + \angle GFI = 180° \quad \rightarrow \quad \boldsymbol{\angle GFI = 47°}$$



Since F is the intersection of segments GJ and IH, angles opposite each other **at F are vertical angles** and thus **congruent**.
So, the angle opposite $\angle GFI = 47°$ in triangle HFJ is **also $\boldsymbol{\angle HFJ = 47°}$**.

In triangle HJF, use the triangle sum theorem and **substitute $\boldsymbol{\angle HFJ = 47°}$**:

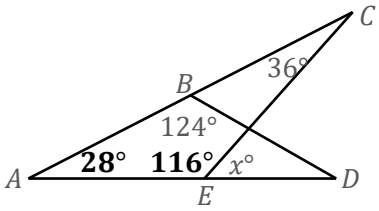
$$\mathbf{53° + y° + \angle HFJ = 180°} \quad \rightarrow \quad 53° + y° + \mathbf{47°} = 180 \quad \rightarrow \quad 100° + y° = 180° \quad \rightarrow \quad \boldsymbol{y° = 80°}$$

Therefore, **the value of $\boldsymbol{y}$ is 80** and the correct answer is **B**.

013. ***Explanation and Answer***

Since $AB = BD$, **triangle ABD is isosceles**, so the base angles, **∠BAD and ∠BDA, are congruent**. And the sum of angles in a triangle is 180°. So,

$$\mathbf{124° + \angle BAD + \angle BDA = 180°} \quad \rightarrow \quad 124° + \mathbf{2}\angle \boldsymbol{BAD} = 180°$$
$$\rightarrow \quad 2\angle BAD = 56°$$
$$\rightarrow \quad \boldsymbol{\angle BAD = 28°}$$



Use angle sum of triangle for △ACE:

$$\mathbf{28° + 36° + \angle AEC = 180} \quad \rightarrow \quad 64° + \angle AEC = 180° \quad \rightarrow \quad \boldsymbol{\angle AEC = 116°}$$

Consider **∠AEC and ∠CED (x°) are linear pair**:

$$\angle AEC + \angle CED = 180° \quad \rightarrow \quad \mathbf{116° + x° = 180°} \quad \rightarrow \quad \boldsymbol{x° = 64°}$$

Therefore, the **value of $\boldsymbol{x}$ is 64** and the correct answer is **C**.

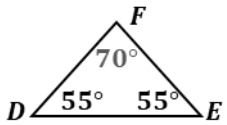
014. ***Explanation and Answer***

Identify the given information:
Triangle DEF is isosceles, meaning it has **at least two congruent angles**. The measure of **angle $\boldsymbol{D}$ is 55°**.

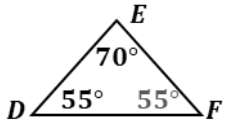
In an **isosceles triangle**, the **two base angles are congruent**. This means that either angle **D is one of the two equal base angles**, or it is **the unique vertex angle**.

Case analysis for possible measures of **angle E**:

Case 1: $\boldsymbol{\angle D = \angle E = 55°}$:
In this case, **$\boldsymbol{\angle D}$ and $\boldsymbol{\angle E}$ are the two congruent base angles** of the isosceles triangle.
Thus, **$\boldsymbol{\angle E}$ could be 55°** in this configuration.



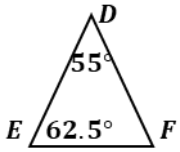
Case 2: $\boldsymbol{\angle D = \angle F = 55°}$:
In this case, **$\boldsymbol{\angle D}$ and $\boldsymbol{\angle F}$ are the two congruent base angles** of the isosceles triangle.
The measure of the third angle, $\angle E$, would then be:
$\boldsymbol{\angle E} = 180° - 55° - 55° = \mathbf{70°}$
Thus, **$\boldsymbol{\angle E}$ could be 70°** in this configuration.



Case 3: $\boldsymbol{\angle D = 55°}$ **and** $\boldsymbol{\angle E = \angle F}$:
Here, **$\boldsymbol{\angle D}$ is the unique vertex angle**, and $\angle E$ and $\angle F$ are the two congruent angles.
Setting up the equation for the sum of the angles:

$$\mathbf{55° + \angle E + \angle F = 180°} \quad \rightarrow \quad 55° + 2\angle E = 180° \quad \rightarrow \quad 2\angle E = 125° \quad \rightarrow \quad \boldsymbol{\angle E = 62.5°}$$

Thus, **$\boldsymbol{\angle E}$ could also be 62.5°**.



Therefore, based on the cases, **$\boldsymbol{\angle E}$ can measure 55°, 62.5°, or 70°** and the correct answer is **D**.

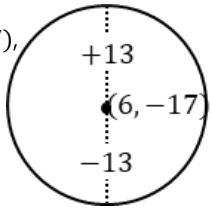
Add 25 and 121 to both sides and rewrite the equation in standard form:

$$x^2 + 10x + y^2 - 22y = -46 \quad \rightarrow \quad x^2 + 10x + \mathbf{25} + y^2 - 22y + \mathbf{121} = -46 + \mathbf{25} + \mathbf{121}$$
$$\rightarrow \quad (\boldsymbol{x+5})^{\mathbf{2}} + (\boldsymbol{y-11})^{\mathbf{2}} = \mathbf{100}$$

Therefore, the center of the circle is $(\mathbf{-5, 11})$ and the correct answer is **B**.

035. ***Explanation and Answer***

The standard form of a circle's equation is $(\boldsymbol{x-h})^{\mathbf{2}} + (\boldsymbol{y-k})^{\mathbf{2}} = \boldsymbol{r}^{\mathbf{2}}$, where $(\boldsymbol{h, k})$ **is the center**, and $\boldsymbol{r}$ **is the radius**. From the given equation, $(x-6)^2 + (y+17)^2 = 169$, the **center is at** $(\mathbf{6},\ \mathbf{-17})$, and the **radius is** $\sqrt{\mathbf{169}} = \sqrt{\mathbf{13^2}} = \mathbf{13}$.

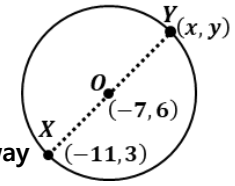


Determine the possible $\boldsymbol{y}$**-values** $(\boldsymbol{b})$ **of points** on the circle:
The $\boldsymbol{y}$**-coordinate** ranges from $\mathbf{-17 + 13}$ to $\mathbf{-17 - 13}$, giving a possible range of $\mathbf{-4}$ **to** $\mathbf{-30}$.

Therefore, the correct answer is **A**.

036. ***Explanation and Answer***

The equation $(x+7)^2 + (y-6)^2 = 25$ is a circle in the standard form $(x-h)^2 + (y-k)^2 = r^2$, where (h, k) is the center of the circle, and r is the radius. From the given equation, the **center** of the circle **is** $\boldsymbol{O}$ $(\mathbf{-7, 6})$, and the radius is $\sqrt{25} = 5$.



Point $\boldsymbol{X}$ $(\mathbf{-11, 3})$ **lies on the circle** and we are told that $\boldsymbol{XY}$ **is the diameter** of the circle, which means **both** $\boldsymbol{X}$ **and** $\boldsymbol{Y}$ **lie on opposite ends of the circle** with the **center** $\boldsymbol{O}$ $(\mathbf{-7, 6})$ **exactly midway between them**.

The **midpoint of any diameter** of a circle **is the center of the circle**. So, the center point $\boldsymbol{O}$ $(\mathbf{-7, 6})$ **is the midpoint** between points X $(-11, 3)$ and Y (x, y). Using the **midpoint formula**:

$$\boldsymbol{M} = \left(\frac{\boldsymbol{x_1 + x_2}}{\mathbf{2}}, \frac{\boldsymbol{y_1 + y_2}}{\mathbf{2}}\right) \quad \rightarrow \quad M = \left(\frac{\boldsymbol{x} + (\mathbf{-11})}{2}, \frac{\boldsymbol{y} + \mathbf{3}}{2}\right) \quad \rightarrow \quad M = \left(\frac{x-11}{2}, \frac{y+3}{2}\right) = (\mathbf{-7, 6})$$

Solving for x and y:

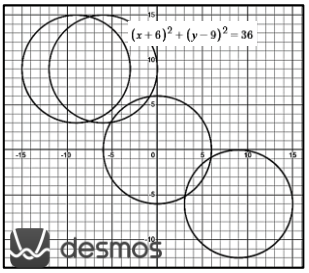
$$\frac{\boldsymbol{x - 11}}{\mathbf{2}} = \mathbf{-7} \quad \rightarrow \quad x - 11 = -14 \quad \rightarrow \quad \boldsymbol{x = -3}$$
$$\frac{\boldsymbol{y + 3}}{\mathbf{2}} = \mathbf{6} \quad \rightarrow \quad y + 3 = 12 \quad \rightarrow \quad \boldsymbol{y = 9}$$

Therefore, the **coordinates of point** $\boldsymbol{Y}$ **are** $(\mathbf{-3, 9})$ and the correct answer is **A**.

037. ***Explanation and Answer***

The $\boldsymbol{y}$**-axis** contains points of the form $(\mathbf{0},\ \boldsymbol{b})$. For a circle to touch the y-axis at exactly one point, the **line** connecting the **center** of the circle, $(\boldsymbol{h},\ \boldsymbol{k})$, **to** the point of tangency $(\mathbf{0},\ \boldsymbol{b})$ **must be parallel to the** $\boldsymbol{x}$**-axis**. This means that the y-coordinates of the center and the point of tangency are the same, so $\boldsymbol{k = b}$. Furthermore, the **distance between the center** of the circle **and the point** on the y-axis **must equal the absolute value of the** $\boldsymbol{x}$ **−coordinate of the center,** $|\boldsymbol{h}|$. Thus, **the distance** between the center of the circle and the y-axis **is the absolute value of the** $\boldsymbol{x}$ **−coordinate** of the center, $|\boldsymbol{h}|$, and this distance **must equal** the **radius** of the circle for there to be exactly one intersection point on the y −axis.

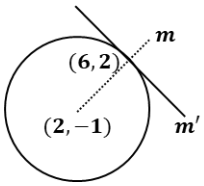
Let's analyze the equation: $(x+6)^2+(y-9)^2=36$
Here, the **center** of the circle is $(-6,\ 9)$, and the **radius is** $\sqrt{36}=6$. To check if this circle intersects the y −axis at exactly one point, we set $x\ =\ 0$ and **check the distance** from the center to the y −axis: $|\ h\ |=|\ -6\ |=\ 6$
Since the **radius of the circle** $(r=6)$ **equals the absolute value of the** x **−coordinate** of the center $(|-6|=6)$, the circle intersects the y −axis at exactly one point.
Or use DESMOS.



Therefore, the correct answer is **C**.

038. ***Explanation and Answer***

The given circle has a center at $(2,-1)$ and a radius passing through the point $(6,2)$. The **tangent line** l **is perpendicular to the radius of the circle at the point of tangency**, which is $(6,2)$.



First, calculate the **slope** (m) **of the radius** that connects the center $(2,-1)$ to the point of tangency $(6,3)$:

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{2-(-1)}{6-2}=\frac{3}{4}$$

Since the **tangent line** l **is perpendicular to the radius**, its slope (m') will be the **negative reciprocal** of the slope of the radius:

$$m'=-\frac{1}{m}=-\frac{4}{3}$$

Write the Equation of the Tangent Line:
Using the slope-intercept form of a line equation with the point $(6,2)$:

$$y=mx+b\ \ \rightarrow\ \ y=-\frac{4}{3}x+b.$$

Substitute $(6,2)$ for x and y:

$$y=-\frac{4}{3}x+b\ \ \rightarrow\ \ 2=-\frac{4}{3}(6)+b\ \ \rightarrow\ \ b=10$$

So, the line equation is

$$y=-\frac{4}{3}x+10.$$

Substituting each coordinate to the equation:

$$y=-\frac{4}{3}x+10\ \ \rightarrow\ \ 6=-\frac{4}{3}(3)+10\ \ \rightarrow\ \ 6=6$$

Therefore, the correct answer is **C**.

039. ***Explanation and Answer***

A **tangent line** to a circle at a point **is perpendicular to the radius** of the circle drawn to that point. This means that the **slope of the tangent line is the negative reciprocal of the slope of the radius**.

The radius of the circle is the **line segment** connecting the center of the circle $(-7,-5)$ to the point of tangency $(-9,-9)$. The formula for the slope between two points $(x_1,\ y_1)$ and (x_2,y_2) is: $slope\ of\ radius, m=\frac{y_2-y_1}{x_2-x_1}$

Substituting the coordinates:

$$m=\frac{y_2-y_1}{x_2-x_1}\ \ \rightarrow\ \ m=\frac{-5-(-9)}{-9-(-7)}\ \ \rightarrow\ \ m=\frac{4}{-2}=-2$$

Since the tangent line is perpendicular to the radius, the **slope of the tangent line,** m'**, is the negative reciprocal of the slope of the radius**. The negative reciprocal of -2 is:

$$slope\ of\ line\ t, m'=-\frac{1}{m}\ \ \rightarrow\ \ m'=-\frac{1}{-2}=\frac{1}{2}$$

Therefore, the slope of line t is $\frac{1}{2}$ and the correct answer is **A**.