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18. (2023 UKMT Junior question)

Two numbers, x and y, satisfy the conditions $\mathbf{0} < \boldsymbol{x} < \boldsymbol{y} < \mathbf{1}$.

Which is the largest and smallest of these expressions?

$$\frac{x}{y} \qquad x - y \qquad \frac{x + y}{2} \qquad \frac{y}{x} \qquad y - x$$

Solving Strategy

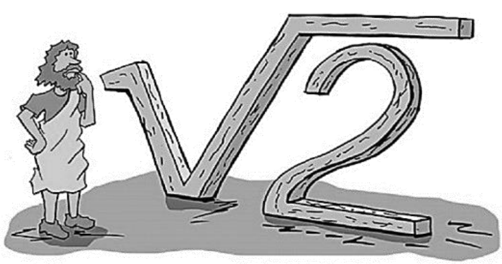
Understanding the properties of inequalities and basic algebraic manipulations are key to solving this problem. It's important to keep in mind that for any two numbers, the greater number divided by the smaller one will be larger than 1, while the smaller number divided by the greater one will be less than 1. This principle is fundamental in determining which of the given expressions is the largest.

19. (UKMT Junior similar question)

Consider the following three statements.

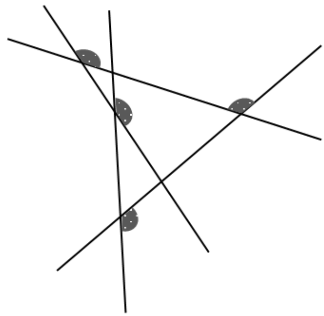
ⓐ Doubling a positive number always makes it larger.
ⓑ Cubing a positive number always makes it larger.
ⓒ Taking the positive square root of a number always makes it smaller.

Which statements are true?



48. (2023 UKMT Junior question)

What is the total sum of the four indicated angles in the figure?

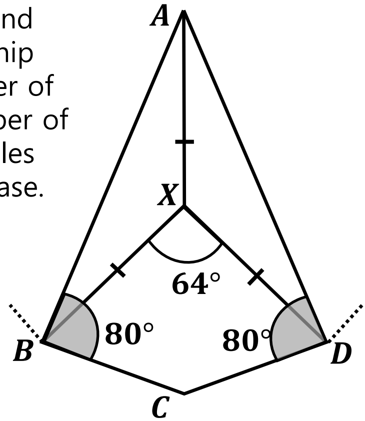


Solving Strategy

The key to solving this problem involves understanding the properties of angles in a triangle and on a straight line, specifically the External Angle Theorem. The approach requires assigning variables to the angles and forming equations based on the relationships between the angles. The final step involves adding the equations to find the sum of the marked angles.

64. (UKMT Junior similar question)

In regular polygons, the sides and angles have a specific relationship that is influenced by the number of sides. For instance, as the number of sides increases, the interior angles of a polygon also tend to increase.

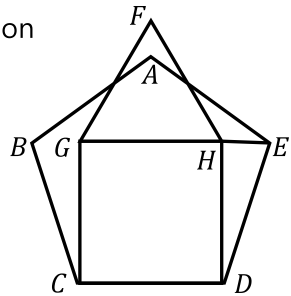


In the diagram BC and CD are sides of a regular n-sided polygon, $\angle ABC = \angle ADC = 80°$, $\angle BXD = 64°$ and $AX = BX = DX$.

What is the value of n?

65. (UKMT Junior similar question)

The diagram shows a regular pentagon ABCDE, a square CDHG and an equilateral triangle FGH.

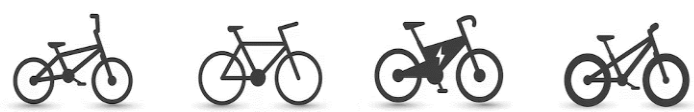


What is the size of angle FHE?

78. (UKMT Junior similar question)

The distance between Jeju and Donomi is 205 kilometers. Minny left Jeju at 11:00 on Friday for Donomi. Kay left Donomi for Jeju at 12:00 the same day. They travelled on the same road. Up to the time when they met, Minny's average speed was 30 kilometers per hour, and Kay's average speed was 40 kilometers an hour.

At what time did Minny and Kay meet?



79. (2022 UKMT Junior question)

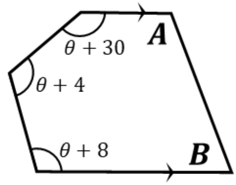
Bob's Bookstore holds fewer than 200 books. All the books are in blue, green, red or white covers. The ratio of blue books to green books is 1 : 3, the ratio of green books to red books is 2 : 5, and the ratio of red books to white books is 4:5.

How many books are there in Bob's bookstore?

Solving Strategy

The key to solving this problem lies in understanding and applying the concept of ratios properly. By expressing all quantities in terms of one variable, it allows us to form an equation and solve for this variable. This strategy is particularly useful in problems involving proportionality and ratios.

52. *Solution & Answer*



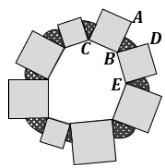
The sum of the interior angles of a pentagon is 540°. The two parallel sides of the pentagon form included angles, and the sum of included angles is 180°.
($A + B = 180°$)

Subtracting the values from the total sum of interior angles, we get the sum of the remaining three angles as 360°.
Therefore, we can set up the equation;

$$\theta + 30 + \theta + 4 + \theta + 8 = 360$$
$$3\theta + 42 = 360$$
$$3\theta = 318 \quad \rightarrow \quad \theta = 106°$$

Therefore, the value of θ is **106**°.

53. *Solution & Answer*



Let's denote the sum of the marked angles as **X**.
To determine X, we **analyse one vertex** of the octagon, specifically vertex B.

The vertex B is shared by two squares, hence the **∠ABC** and **∠DBE** are angles of a square, each measuring **90**°.
So, the equation for the **sum of the angles at vertex B** becomes:

∠CBE + ∠ABD + 90 + 90 = 360°
∠CBE + ∠ABD = 180°
∠ABD = 180 - ∠CBE

A similar equation can be derived for each of the eight marked angles around the octagon. By adding up these eight equations, the sum of the marked angles (X) becomes:

X=8×180°-the sum of the interior angles of the octagon

The sum of the interior angles of a polygon with $\boldsymbol{n}$ vertices is $(n - 2) \times 180°$, so for an octagon (n=8), this sum equals $(8 - 2) \times 180°$.

Substituting this value into the equation for X gives us:

$$\boldsymbol{X = 8 \times 180 - (8 - 2) \times 180 = 360°}$$

Therefore, the sum of the eight exterior marked angles is **360**.

54. Solution & Answer

The problem involves an isosceles triangle ABC, where AB equals AC. This implies that **∠ABC = ∠ACB** due to the properties of an isosceles triangle. By setting the expressions for these two angles equal to each other:

$$4y - 15 = 3y - 7 \quad \rightarrow \quad \boldsymbol{y = 8}$$

Substituting **y=8** into the expressions for ∠ABC and ∠ACB gives us 4(8)-15°=3(8)-7°=17°. Knowing that the sum of the angles in a triangle is 180°, we can find the third ∠BAC, by subtracting the known angles from 180°, giving us 180°-17°-17° = 146°.

Setting the expression for ∠BAC equal to 146° allows us to solve for x:

$$5x + 26 = 146 \quad \rightarrow \quad 5x = 120 \quad \rightarrow \quad \boldsymbol{x = 24}$$

Therefore, the value of $x + y$ is 24+8=**32**.

55. Solution & Answer

The provided problem involves a right-angled triangle, and we are tasked with determining the values of a and b for the three angles that are expressed in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

Given that the triangle is right-angled, so the angle (6a + 4b)° is the right angle, which gives us the equation:

$$\boldsymbol{6a + 4b = 90}$$

The sum of the angles in any triangle is 180 degrees. Therefore, the sum of the remaining two angles, (7a - b)° and (11a - 2b)°, must also equal 90 degrees. This provides us with a second equation: 7a - b + 11a - 2b = 90, which simplifies to:

$$\boldsymbol{18a - 3b = 90}$$

We now have a system of two equations:

6a + 4b = 90 and 18a - 3b = 90

By multiplying the first equation by 3 and subtracting the second equation from the result, we get:

$$\boldsymbol{15b = 180} \quad \rightarrow \quad \boldsymbol{b = 12}$$

Substituting **b=12** into the first equation gives us 6a+4(12)=90, which further simplifies to $\boldsymbol{a = 7}$.

Therefore, the product of a and b is $7 \times 12 = \mathbf{84}$.

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05. (2023 UKMT Junior question)

Grandpa gave away his entire collection of vintage watches to three people. His son received 8 more than a third of the total; his daughter received 8 more than a third of what was then left; finally his friend received 8 more than a third of what was then left.

What is the sum of the digits of the number of watches which were in Grandpa's collection?

Solving Strategy

The key in solving this problem lies in understanding how fractions work when subtracting or dividing numbers from them. It requires setting up an algebraic equation based on information given about how many watches were distributed at each step.

07. (2023 UKMT Junior question)

Charles was born in this century. On his birthday in 2025, his age was equal to the sum of the digits of the year in which he was born.

In which of these years will his age on his birthday be twice as much as the sum of the digits of that specific year?

Solving Strategy

Firstly, define variables for unknowns and set up equations based on provided conditions about relationships between person's current or future ages and sums of corresponding years' digits. Secondly, solve these equations considering constraints such as possible values for each digit.

15. (2022 UKMT Junior question)

John and David engage in a game where each participant starts with 15 cards. In each round of the game, one player wins and is given 3 cards; and his opponent has 1 card removed. At the end of the game, John and David have 48 cards and 20 cards respectively.

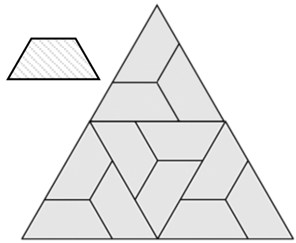
How many rounds of the game did John win?

Solving Strategy

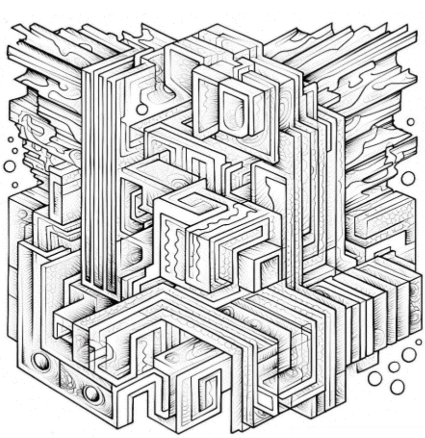
First, observe the net gain of cards in each round and calculate the total number of rounds by comparing the initial and final total number of cards. Then, set up an equation reflecting the number of cards John gains and loses in the rounds he wins and loses respectively. Solving this equation will give you the number of rounds John wins.

62. (UKMT Junior similar question)

The small trapezium on the right has three equal sides and angles of 60° and 120°. Twelve copies of this trapezium can be arranged to form a large triangle, as shown. The large triangle has perimeter 36cm.



What is the perimeter of the small trapezium?



76. (2021 UKMT Junior question)

In 1770, a mathematician named Joseph-Louis Lagrange proved that every positive integer can be written as the sum of four squares.
For instance, 13 can be expressed as $0^2+0^2+2^2+3^2$.

Now, considering this, how many of the first 20 positive integers can be written as the sum of three squares?

Solving Strategy

The key to solving this problem is understanding the properties of squares and knowing how to systematically generate combinations. Start by identifying all the squares less than or equal to the largest number in the given range. Then, systematically add these squares in groups of three to find all possible totals within the range.

79. (2023 UKMT Junior question)

Every cell in the crossnumber puzzle should be filled with a single digit.

across	**down**
1. a cube	1. a prime
2. a square	

What could possibly be the sum of the four digits in the crossnumber?

Solving Strategy

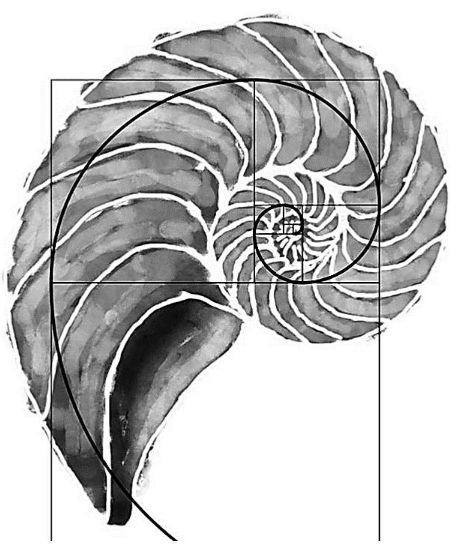
Understanding the properties of cubes, squares, and prime numbers is crucial for solving this problem. It's also important to recognize that there are limited options for two-digit cubes and squares and for primes with certain tens digits.

85. (UKMT Junior similar question)

In mathematics, there's a famous sequence known as the Fibonacci sequence. There are many variations of this sequence, and they can be just as interesting.

Jessica writes down a sequence of six numbers. The rule she uses is, “after the first three terms, each term is the sum of the three previous terms.”
Her sequence is —, —, —, 11, 22, 38.

What is her first term?



13. *Solution & Answer*

Let's suppose Charlie has x ducks. Since he has four times as many rabbits as ducks, he has $4x$ rabbits. Therefore, the total number of animals Charlie has is $\mathbf{5x}$, which is the **sum of x ducks and 4x rabbits.**

Similarly, let's assume Helen has y rabbits. As she has three times as many ducks as rabbits, she has $3y$ ducks. So, the total number of animals Helen has is $\mathbf{4y}$, which is the **sum of y rabbits and 3y ducks.**

The problem states that the total number of animals they both have is 26. This can be represented as the equation:

$$\mathbf{5x + 4y = 26}$$

Here, **x and y are non-negative integers** according to the context of the problem. From the equation,

$$5x \leq 26 \quad \rightarrow \quad x \leq 5$$

So, the only possible values for $\boldsymbol{x}$ are **0, 1, 2, 3, 4 and 5**.

By substituting these values of x into the equation, when **x=2** does the value of **y become a positive integer**.

Let's substitute $\boldsymbol{x} = \mathbf{2}$ into the equation:

$$5(2) + 4y = 26 \quad \rightarrow \quad \boldsymbol{y} = \mathbf{4}.$$

So, with $x = 2$, Charlie has $4 \times 2 = \mathbf{8}$ rabbits, and with $y = 4$, Helen has **4** rabbits.

Therefore, there are a total of 8+4=**12 rabbits** among the animals.

14. *Solution & Answer*

The **value of three 5 kon coins is 15 kon**. In order to form a total of **17 kon using only three coins**, the value of the third type of coin must **be greater than 5 kon** by the remainder of 17 kon after subtracting 15 kon, which **is 2 kon**. Hence, the value of the third type of coin must **be 7 kon**.

This conclusion can be validated by the other given total, 11 kon.

2 kon + 2 kon + 7 kon = 11 kon

Similarly, it is also possible to form 17 won with three coins by using two 5 won coins and one 7 won coin.

5 kon + 5 kon + 7 kon = 17 kon

Therefore, the value of the third type of coin in New Fourland is indeed **7 kon**.

15. ***Solution & Answer***

In order to solve this problem, we need to understand a few basic principles of the game. The first one is that John and David collectively **have a net gain of 2 cards after each round**. This is because the **winner gains 3 cards and the loser loses 1 card**, resulting in a net gain of 2 cards.

At the start of the game, the total number of cards is;

15+15 = **30** cards

After the game ends, the total number of cards is:

48+20 = **68** cards

Since the **net gain** of cards in each round is **2**, the total number of rounds would be;

68-30 = **38 net** gaining cards

38 ÷2 = **19 rounds**

Assuming that John wins '$\boldsymbol{x}$' number of rounds, in the rounds that John **wins**, he gains $\boldsymbol{3x}$ cards, but in the rounds he **loses**, he loses $\boldsymbol{19 - x}$ cards.

Given that John **starts with 15** cards and **ends with 48** cards, we can set up the equation:

$$15 + 3x - (19 - x) = 48$$

$$4x = 52 \quad \rightarrow \quad x = 13$$

Therefore, John wins **13** rounds of the game.

16. ***Solution & Answer***

Let's denote the number of games Sally won as **S**, the number of games Tomo won as **T**, and the number of drawn games as **D**.
And, Sally wins twice as many games as Tomo, so **S=2T**.

The points gained from the won games are **3S for Sally** and **3T for Tomo**. And the points gained from the drawn games are **D for both Sally and Tomo**.
Sally ends with 24 points, and Tomo ends with 15 points, so we can set up the following system of equations:

3S + D = 24 (Sally's total points) ①

3T + D = 15 (Tomo's total points) ②

Using S=2T

3(2T)+D=24 → **6T+D=24** ③

From ③-①

3T=9 → T=3 and D=6, S=6

Therefore, there were 6(S)+6(D)+3(T)=**15** games in the match, Sally **won 6**(S) games, **drew 6**(D) games, and **lost 3**(T) games.

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07. (2023 UKMT Junior question)

The positive integers from 1 to 9 inclusive are placed in the grid, with each cell containing one integer. So that the product of the three numbers in each row or column is as shown.

	?		96
			105
			36
24	126	120	

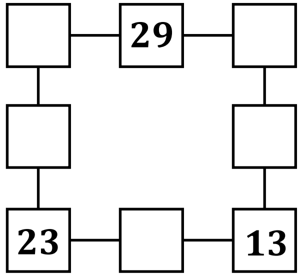
What is the appropriate number to be placed in the top middle cell?

Solving Strategy

This problem requires a firm understanding of factorisation and the properties of integers. It involves evaluating the given products and identifying unique factors that allow you to place certain integers in specific cells. The key strategy is to identify which products have unique factors and use these factors to place the integers. In this case, the unique factors are 5 and 7 which only appear in the products 105 and 56, and 105 and 180 respectively.

21. (2023 UKMT Junior question)

When this prime number square is completed, the eight small squares contain eight different prime numbers, and each of the four sides has total 43.



What is the value of a, b, c, d and e?

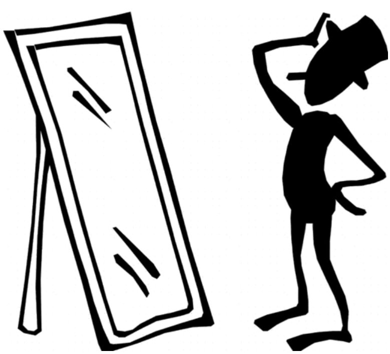
Solving Strategy

The primary strategy to solve this problem lies in understanding the properties of prime numbers and the method of systematic equation solving. First, acknowledging that prime numbers are integers greater than 1 with only two distinct positive divisors, 1 and the number itself. Second, the approach involves setting up and solving a system of equations based on the information given in the problem.

37. (UKMT Junior similar question)

During a mathematics lesson, the teacher introduced a unique activity to her class. She held up a mirror to written numbers, showing that a number like 25 could be seen as 52 when reflected. She explained that this could only happen with certain digits, but not with others.

How many three-digit numbers reflect to a three-digit number?



41. (2023 UKMT Junior question)

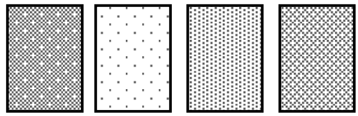
In a football match, Tigers defeated Lions 4-3. The only time Tigers were in the lead was after they scored the final goal.

How many possible half-time scores could there be?

Solving Strategy

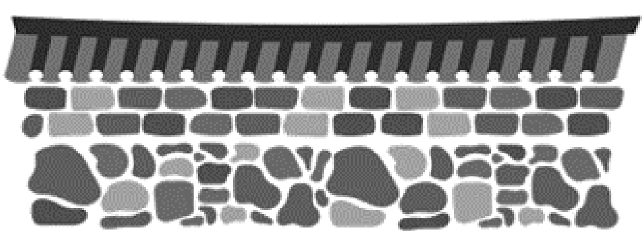
To solve this problem, understanding the scoring progression in a football match and the condition that Tigers could not lead until the final goal is crucial. Logic and reasoning are applied to consider all possible half-time scores under these conditions.

42. (UKMT Junior similar question)



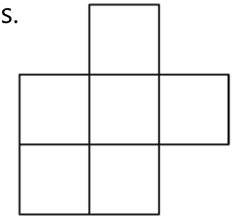
A section of a wall is planned to be decorated with a row of four rectangular tiles.
Three different colours of tiles are available and there are at least two tiles of each colour available. Tiles of all three colours must be used.

How many different ways are there to select the row of four tiles?



47. (2020 UKMT Junior question)

Jennifer possesses a grid containing six cells. She wants to colour two of the cells black so that the two black cells share a vertex but not a side.



How many ways are there for her to accomplish this?

Solving Strategy

This problem requires spatial reasoning and counting principles. The approach involves identifying each vertex and determining the number of pairs of cells that can be colored according to the rules. Summing these possibilities across all vertices gives the total number of ways.

51. (2023 UKMT Junior question)

In the addition shown,
A and B represent different single digits.

$$\begin{array}{r} 7\,7\,A \\ 6\,B\,A \\ +\ B\,B\,A \\ \hline 1\,A\,A\,7 \end{array}$$

What is the value of A-B?

Solving Strategy

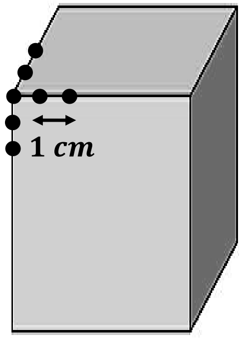
The initial focus on the units column exploits the fact that A is a single digit, and its value is deduced to be 9. Recognizing the carry from the units to the tens column and understanding the constraints of the tens column equation narrows down the possibilities for B to be 0 or 5. Further analysis rules out B=0, leading to the conclusion that B=5. The final step involves substituting these values into the addition and confirming the correctness of the sum.

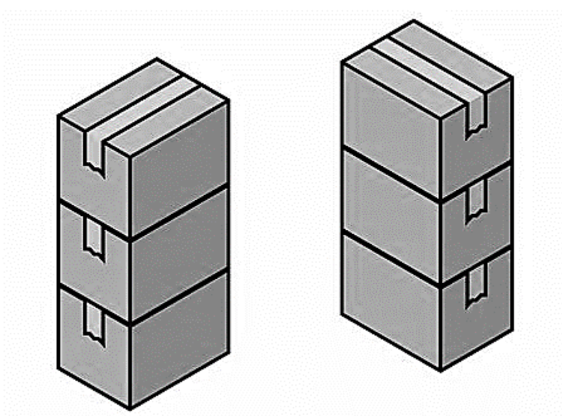
71. (UKMT Junior similar question)

The dimensions of the given cuboid are a width of 10 cm, a length of 10 cm, and a height of 16 cm.

Dots are positioned at 1 cm intervals along every edge, beginning from the vertices. A part of this pattern is shown.

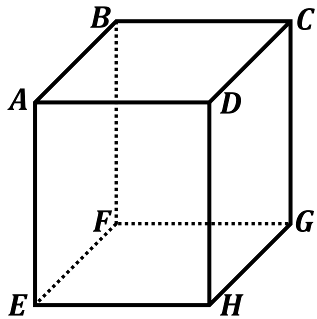
How many dots will there be in total, once the pattern has been completed?





75. (2022 UKMT Junior question)

A triangular pyramid with vertices A, C, D, and H is removed from the shown solid cuboid.



What is the number of edges on the solid that results?

Solving Strategy

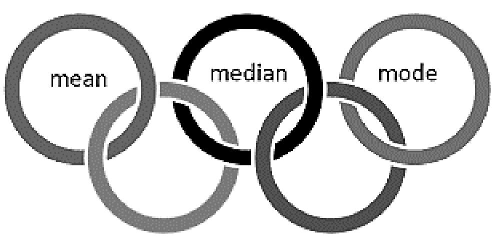
Understand the properties of the shapes involved. In this case, it's important to remember that a cube has 12 edges. Also, note that the removal of a triangular pyramid from a cube results in a new shape but does not change the total number of edges.

79. (UKMT Junior similar question)

The mean is the sum of all numbers divided by the count of numbers. The median is the middle number in an ordered list. The mode is the value that appears most frequently in a data set.

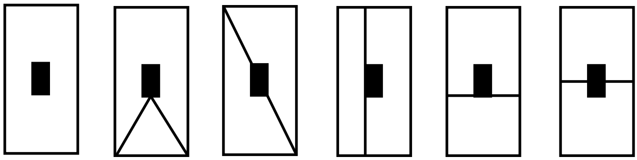
The range of a list of integers is 30, and the median is 17.

What is the smallest possible number of integers in the list?



80. (2020 UKMT Junior question)

Each of these figures is based on a rectangle with a smaller rectangle positioned at the center, as shown.



How many of the figures have rotational symmetry of order two?

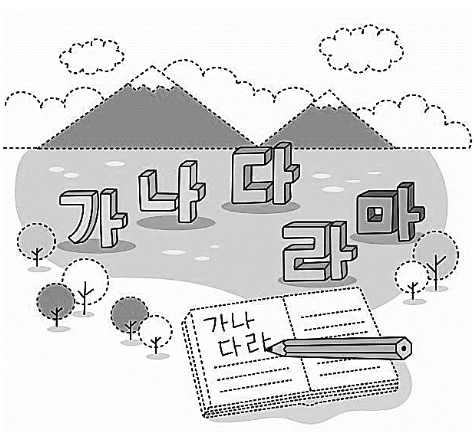
Solving Strategy

To solve this type of problem, it's crucial to understand the concept of rotational symmetry, specifically 'order two', which refers to the object appearing the same after a half turn. Analyzing each figure independently and mentally or physically rotating it by 180° will aid in determining which figures possess this particular form of symmetry.

81. (UKMT Junior similar question)

You have recently been exploring the fascinating world of Korean language and its unique script, Hangul. These symbols, which might appear as mere figures to the unfamiliar eye, each carry a distinct sound in the Korean language. Now, imagine these Hangul consonants as geometrical figures.

How many reflectional symmetries does each of the figures above have?



40. ***Solution & Answer***

The mathematical principle at play here is that to find the total number of integers between two given integers '$\boldsymbol{a}$' and '$\boldsymbol{b}$', inclusive:

$$(\boldsymbol{b}-\boldsymbol{a})+\mathbf{1}$$

Applying this principle to our problem, we subtract -12 from 88 and add 1, which gives us **88-(-12) + 1 = 101**.

Therefore, there are **101** integers.

41. ***Solution & Answer***

The final score of the match was **4-3**, where Tigers defeated Lions. However, Tigers were only leading after they scored their final goal.

Let's consider two scenarios:
The score at **half-time was also 4-3**. And Tigers were not leading at half-time. In the latter scenario, it's important to note that Tigers could have scored at most 3 goals by half-time, and they could not have scored more goals than Lions.
This is because, as per the problem, **Tigers were only in the lead after they scored their final goal**.

Given these conditions, the possible half-time scores with the Tigers' score stated first are:
0-0, 0-1, 0-2, 0-3
1-1, 1-2, 1-3
2-2, 2-3
3-3, and
4-3

Therefore, there are a total of **11** possible half-time scores.

45. ***Solution & Answer***

Suppose that the colours of the tiles are red, blue, and green.

We need to find the number of different ways to arrange the tiles using **at least one tile** of each of the three colours. To fulfil this condition, we need **two tiles of one colour** and **one tile of each of the other two colours**.

We can split our choice of how to arrange the tiles into a sequence of independent choices, and count the number of different ways in which we can make these choices.

The **first choice** is to decide the **colour of the two tiles with the same colour**. There are **three ways** to choose this colour. Once we have made this choice, we have **no further choice of colours** as we must have one tile of each of the two other colours. We now need to count the number of different ways to place the four tiles in a row after we have chosen their colours.

Suppose we have decided to have **two red tiles**, and hence **one blue tile and one green tile**. We decide how to place these tiles by **first choosing the place for the blue** tile, and **then** the place **for the green** tile. There are **4 choices for** the position of the **blue** tile. Whichever position we choose for the blue tile, there **remain 3 choices** of position **for the green** tile. The **two red** tiles will then fill the **two remaining places**. So, the number of possibilities for arranging two red tiles, one blue tile, and one green tile is;

$$\mathbf{4 \times 3 = 12}$$

Since there are **three possible colours** (red, blue, and green) **for the two tiles** of the same colour, the total number of different ways of arranging the tiles is $\mathbf{3 \times 4 \times 3 = 36}$.

Therefore, there are **36** different ways in which the row of four tiles can be chosen.

81. ***Solution & Answer***

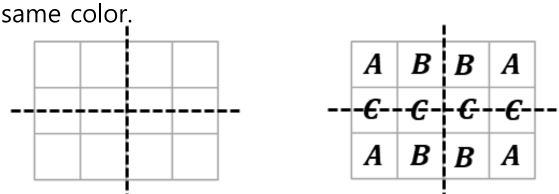
Reflection symmetry, also known as line symmetry or mirror symmetry, is a type of symmetry that involves the property of being unchanged under reflection across a line. This symmetry is present when there is at least one line that acts as a mirror, dividing a figure into two halves such that one half is the mirror image of the other half.

Therefore, the number of the figures that have reflectional symmetry is **3** or **three**.

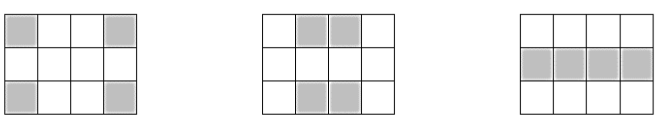
82. ***Solution & Answer***

A shape has reflection symmetry if there is a line going through it which divides it into two pieces that are mirror images of each other.

In Carmen's case, two lines of reflection symmetry are indicated in the grid as dashed lines. From this, we can infer that if the grid with some of the small squares shaded grey possesses both these lines of reflection symmetry, then all four squares labeled 'A' must be of the same color.



Similarly, for maintaining symmetry, all the four squares labeled 'B' must be the same color. The same rule applies to the squares labeled 'C'.
By logically applying these rules of reflection symmetry, Carmen can produce three different grids, as shown in the figure below.



Therefore, there are **3** or **three** different grids.